DISCUSSION PAPER

October 2012 ■ RFF DP 12-42

How Should Benefits and Costs Be Discounted in an Intergenerational Context?

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RESOURCES FOR THE FUTURE

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Abstract

Should governments, in discounting the future benefits and costs of public projects, use a discount rate that declines over time? The argument for a declining discount rate is a simple one: if the discount rates that will be applied in the future are persistent, and if the analyst can assign probabilities to these discount rates, this will result in a declining schedule of certainty-equivalent discount rates. A growing empirical literature estimates models of long-term interest rates and uses them to forecast the declining discount rate schedule. I briefly review this literature, focusing on models for the United States. This literature has, however, been criticized for a lack of connection to the theory of project evaluation. In cost-benefit analysis, the net benefits of a project in year *t* (in consumption units) are to be discounted to the present at the rate at which society would trade consumption in year *t* for consumption in the present. With simplifying assumptions, this leads to the Ramsey discounting formula. The Ramsey formula results in a declining certainty-equivalent discount rate if the rate of growth in consumption is uncertain and if shocks to consumption are correlated over time. Using the extended Ramsey formula to estimate a numerical schedule of certainty-equivalent discount rates is, however, challenging.

Key Words: discount rate, uncertainty, declining discount rate, cost-benefit analysis

JEL Classification Numbers: D61

Contents

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I. Introduction

Many of the decisions we make today have implications for future generations. This includes decisions to invest in physical capital or in research and development of knowledge, as well as decisions that affect environmental capital. Perhaps the most salient example of the latter is the stock of greenhouse gases (GHGs) in the earth‛s atmosphere: GHGs emitted to the atmosphere today will affect the earth‛s climate for generations to come. Internationally, experts debate what fraction of the world‛s resources should be devoted to reducing carbon emissions. In the academic literature, this debate is reflected in arguments about the optimal carbon tax. According to Nordhaus (2007), the optimal carbon tax should currently be less than $10 per ton of carbon dioxide. The Stern Review (Stern 2006) argues for a carbon tax that is 10 times as large. Much of this discrepancy can be attributed to differences in the rate at which the authors discount future climate damages. Stern suggests that a discount rate of 1.4 percent is appropriate; Nordhaus uses a rate closer to 5 percent.

In project analysis, the rate at which future benefits and costs are discounted often determines whether a project passes the benefit–cost test. This is especially true of projects with long horizons, such as projects to reduce GHG emissions. In the United States, the Office of Management and Budget (OMB) recommends that project costs and benefits be discounted at a constant exponential rate, although this rate may be lower for projects that affect future generations (OMB 2003). In contrast, France and the United Kingdom use discount rate schedules, pictured in Figure 1, in which the discount rate applied to benefits and costs in future years declines over time: the rate used to discount benefits in year 50 to the present is lower than the rate used to discount benefits in year 10 to the present. In the United Kingdom, for example,

* Distinguished Professor of Economics, University of Maryland, and Senior Fellow, Resources for the Future. This paper was written for a conference on Sustainable Consumption, held at the University of Manchester in March 2012. I would like to thank Partha Dasgupta, Randy Lutter, Dale Southerton, Alistair Ulph, and Hal Wilhite for comments, and I thank members of the U.S. Environmental Protection Agency‛s National Center for Environmental Economics for helpful discussions.

a rate of 3.5 percent is used to discount benefits in year 10, but a rate of 2.5 percent is used to discount benefits in year 70, and the rate falls to 1 percent in year 300.

Whether the use of a declining discount rate (DDR) is more present-oriented than the use of a constant discount rate depends, of course, on the levels of each. It is clear, however, that the use of a DDR schedule that begins at 4 percent will yield a higher net present value of whatever is being discounted than the use of a constant exponential discount rate of 4 percent. This raises two questions. Should governments, in discounting the future benefits and costs of public projects, use a discount rate that declines over time? How should the discount rate—or discount rate schedule—be set?

This paper focuses on the first question: should governments, in discounting the future benefits and costs of public projects, use a discount rate that declines over time? To answer this question, I review the arguments for a DDR. The basic argument is a simple one: if the discount rates that will be applied in the future are persistent, and if we can assign probabilities to these discount rates, this will result in a declining schedule of certainty-equivalent discount rates. In the economics literature, this argument was first advanced by Weitzman (1998, 2001) in an expected net present value (ENPV) framework. If an analyst is evaluating the net present value of the benefits of a project using a constant exponential discount rate, and if the discount rate is uncertain, taking the expected value of the discount factor will yield a declining certainty-equivalent discount rate. This has led to a growing empirical literature that estimates reduced-form models of interest rate determination and their implications for the shape of the DDR schedule. I briefly review this literature, focusing on models of interest rate determination for the United States and their implications for calculating the marginal social cost of carbon—the present value of damages from emitting an additional ton of carbon.

The ENPV literature has, however, been criticized for a lack of connection to the economics literature on project evaluation. In benefit–cost analysis, the net benefits of a project in year t (in consumption units) are to be discounted to the present at the rate at which society would trade consumption in year t for consumption in the present. This approach leads to the Ramsey discounting formula, in which the discount rate applied to net benefits at time t, ρ_t, equals the sum of the utility rate of discount (δ) and the rate of growth in consumption between t and the present (g_t), weighted by the elasticity of marginal utility of consumption (η).[1] The

[1] Formally, $\rho_t = \delta + \eta \cdot g_t$.

Ramsey formula can also lead to a DDR if the rate of growth in consumption (g_t) is uncertain and if shocks to consumption are correlated over time. Uncertainty about the mean and variance of the rate of growth in consumption can also lead to a declining certainty-equivalent discount rate.

The paper is organized as follows. Section II discusses the (ENVP) approach and summarizes empirical estimates of certainty-equivalent discount rates for the United States. Section III focuses on the consumption rate of discount as represented by the Ramsey formula. I discuss conditions under which uncertainty in the rate of growth in consumption, as reflected in the extended Ramsey formula, leads to a declining certainty-equivalent discount rate. Section IV addresses the issue of how, empirically, the parameters of the Ramsey formula might be estimated. Section V summarizes the arguments for using a DDR in project analysis and discusses the difficulties in determining empirically how the discount rate schedule should be set.

II. The Expected Net Present Value Approach to Discounting

In performing a cost–benefit analysis, an analyst must typically discount a stream of net benefits to the present. If Z(t) denotes net benefits at time t and net benefits are discounted at a constant exponential rate r, Z(t)exp(-rt) is the present value of net benefits at time t.[2] If the discount rate r is uncertain, then the expected value of net benefits is given by

$$A(t)Z(t) = E(exp(-rt))Z(t) \tag{1}$$

where expectation is taken with respect to r. *A(t)* is the expected value of the discount factor and $R_t \equiv -(dA_t/dt)/A_t$ is the certainty-equivalent discount rate. If the probability distribution over r is stationary, then, because the discount factor is a convex function of r, the certainty-equivalent discount rate, R_t, will decline over time (Weitzman 1998, 2001).[3]

This is illustrated in Table 1, which contrasts the present value of £1,000 received at various dates using a constant discount rate of 4 percent versus a constant discount rate that equals 1 percent and 7 percent with equal probability. Jensen‘s inequality guarantees that the

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[2] I assume that Z(t) represents certain benefits. If benefits are uncertain, I assume that they are uncorrelated with r and that Z(t) represents certainty-equivalent benefits.

[3] Gollier and Weitzman (2010) discuss the theoretical underpinnings for the ENPV approach. The approach is consistent with utility maximization in the case of a logarithmic utility function.

present value computed using the mean discount rate of 4 percent is always smaller than the expected value of the discount factor. That is,

$$E(exp(-rt)) > exp(-E(r)t)) \tag{2}$$

This effect is magnified as t increases, implying that R_t declines over time.

This was first pointed out in the context of intergenerational discounting by Weitzman (1998, 2001). In a seminal article, ―Gamma Discounting," Weitzman (2001) showed that, if r follows a gamma distribution with mean μ and variance ζ^2, the certainty-equivalent discount rate is given by

$$R_t = \mu/[1 + t\sigma^2/\mu] \tag{3}$$

The gamma distribution provides a good fit to the responses Weitzman obtained when he asked more than 2,000 Ph.D. economists what rate should be used to discount the costs and benefits associated with programs to mitigate climate change. The associated mean (4 percent) and standard deviation (3 percent) of responses lead to the schedule of certainty-equivalent discount rates in Table 2.

The declining certainty-equivalent discount rate in Gamma Discounting follows directly from Jensen's inequality and the fact that the distribution over the discount rate is constant over time. In the more general case in which the discount rate varies over time, so that

$$A(t) = E[exp(-\textstyle\sum_{\tau=1\ldots t} r_\tau)] \tag{4}$$

the shape of the R_t path depends on the distribution of the $\{r_\tau\}$. If $\{r_\tau\}$ are independently and identically distributed, the certainty-equivalent discount rate is constant. In equation (4), there must be persistence in uncertainty about the discount rate for the certainty-equivalent rate to decline. If, for example, shocks to the discount rate are correlated over time, as in equation (5)

$$r_t = \pi + e_t \text{ and } e_t = ae_{t-1} + u_t\,, |a| \le 1 \tag{5}$$

the certainty-equivalent discount rate will decline over time (Newell and Pizer 2003).[4]

[4] In equation (5), the interest rate follows an AR(1) process. In estimating (5), one typically assumes that $\pi \sim N(\mu_\pi, \zeta_\pi^2)$ and $\{u_t\} \sim$ i.i.d. $N(0, \zeta_u^2)$.

This suggests that it is important to consider the underlying source of uncertainty about discount rates. In Gamma Discounting, experts disagree about a constant rate of discount. Weitzman (2001) treats the views of experts as equally valid and uses them to estimate the parameters of a true, underlying distribution of the interest rate. As Freeman and Groom (2010) have shown, viewing the responses of experts as forecasts of the true mean interest rate leads to a method of combining expert responses that differs from the one Weitzman used. When experts' forecast errors are independently and identically distributed, the decline in the certainty-equivalent discount rate is attenuated, compared to Gamma Discounting, because each expert provides additional information that reduces the variance of the true mean interest rate. This is illustrated in Figure 2, which shows how increasing the number of experts whose opinions are aggregated alters the path of the certainty-equivalent discount rate and contrasts the aggregation approaches used by Freeman and Groom (2010) and Weitzman (2001).

One can criticize both Weitzman (2001) and Freeman and Groom (2010) because the authors view the appropriate discount rate as constant. As Dasgupta (2008) has pointed out, this is an especially inappropriate assumption when discounting over long horizons. It is also true that, in the context of Gamma Discounting, the source of the uncertainty that gives rise to a DDR is disagreement among experts. Whether the discount rate represents the return to capital or the consumption rate of discount, it is likely to change over time. And, a more satisfying approach is to model the underlying economic source of the uncertainty.

Empirical Estimates of the DDR Schedule for the United States

The predominant approach followed in the empirical DDR literature is to view r_t as representing the return to risk-free capital at time t, and to develop econometric models to forecast r_t. The empirical DDR literature includes models of interest rate determination for the United States (Newell and Pizer 2003; Groom et al. 2007); Australia, Canada, Germany, and the United Kingdom (Hepburn et al. 2009; Gollier et al. 2008); and France, India, Japan, and South Africa (Gollier et al. 2008). I focus on the empirical DDR literature as applied to the United States. Newell and Pizer (2003) estimate reduced-form models of bond yields for the United States, using two centuries of data on Treasury bonds; they use these models to estimate certainty-equivalent discount rates over the next 400 years. The authors assume that interest rates follow an autoregressive process. This is given by equation (5) in the case of a first-order autoregressive moving average [AR(1)].[5] Equation (5) implies that the mean interest rate is uncertain, and that deviations from the mean interest rate will be more persistent the higher is *a*.

The authors demonstrate that the instantaneous certainty-equivalent interest rate corresponding to (5) is given by

$$R_t = \mu_\pi - t\sigma_\pi^2 - \sigma_u^2 f(a,t) \qquad (6)$$

where $f(a,t)$ is increasing in *a* and *t*. How fast the certainty-equivalent interest rate declines depends on the variance in the mean interest rate as well as the persistence of shocks to the mean interest rate (i.e., on *a*). When $a = 1$, interest rates follow a random walk (RW). To illustrate the implications of persistence, if $a = 1$, $\mu_\pi = 4$ percent, $\sigma_\pi^2 = 0.52$ percent and $\sigma_u^2 = 0.23$ percent, the certainty-equivalent discount rate declines from 4 percent today to 1 percent 100 years from now. In contrast, a value of $a < 1$ (a mean-reverting [MR] model) implies that interest rates will revert to μ_π in the long run. When $a = 0.96$, $\mu_\pi = 4$ percent, $\sigma_\pi^2 = 0.52$ percent and $\sigma_u^2 = 0.23$ percent, the certainty-equivalent discount rate is 4.0 percent today and 3.6 percent 100 years from now (Newell and Pizer 2003).

Newell and Pizer use results from their preferred specifications of RW and MR models to simulate the path of certainty-equivalent discount rates.[6] In the RW model (see Figure 3) the certainty-equivalent discount rate falls from 4 percent today to 2 percent in 100 years; in the MR model, a certainty-equivalent discount rate of 2 percent is reached only in 300 years. The authors cannot reject the RW hypothesis, but investigate the implications of both models for calculating the marginal social cost of carbon.[7] Using damage estimates from Nordhaus (1994), the marginal social cost of carbon is computed as the present discounted value of global damages from emitting a ton of carbon in 2000, discounted at a constant exponential rate of 4 percent and using certainty-equivalent rates from the two models. The marginal social cost of carbon increases from $5.29 using a constant rate of 4 percent to $10.44 (1989 US$) using the RW model.

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[5] The authors estimate autoregressive models in the logarithms of the variables ($\ln r_t = ln\pi + e_t$) to avoid negative interest rates. Their preferred models are AR(3) models in which $e_t = a_1 e_{t-1} + a_2 e_{t-2} + a_3 e_{t-3} + u_t$.

[6] The preferred models are AR(3) models, estimated using the logarithms of the variables (see footnote 2). The RW model imposes the restriction that $a_1 + a_2 + a_3 = 1$.

[7] The point estimate of $a_1 + a_2 + a_3 = 0.976$ with a standard error of 0.11. The authors also note that the MR model, when estimated using data from 1798 through 1899, overpredicts interest rates in the first half of the 20th century.

The subsequent literature, following the literature in Finance, has estimated more flexible reduced-form models of interest rate determination. Groom et al. (2007) estimate five models for the United States using the same data as Newell and Pizer (2003). The first two are RW and MR models identical to those in Newell and Pizer (2003); the third is an autoregressive IGARCH (integrated generalized autoregressive conditional heteroskedasticity) model that allows the conditional variance of the interest rate (held fixed in equation (5)) to vary over time; the fourth is a regime-switching model that allows the interest rate to shift randomly between two regimes that differ in their mean and variance. The final model, which outperforms the others in within- and out-of-sample predictions, is a state-space model. This is an autoregressive model that allows both the degree of mean reversion and the variance of the process to change over time.[8]

Groom et al. (2007) use their estimation results to simulate certainty-equivalent discount rates and use these to compute the marginal social cost of carbon using data from Nordhaus (1994). Figure 4 displays the paths of certainty-equivalent discount rates based on all five models, starting from a discount rate of 4 percent. The certainty-equivalent rates from the state-space model decline more rapidly than rates produced by the RW model (see Figure 4) for the first 100 years, leveling off at about 2 percent. The RW model yields a certainty-equivalent discount rate of 2 percent at 100 years and 1 percent in year 200, declining to about 0.5 percent when t=400. The social cost of carbon is, however, higher using discount rates produced by the state-space model because of an initial rapid decline in the certainty-equivalent rate and the path of climate change damages: $14.44 per ton of carbon using the state-space model vs. $10.32 per ton using the RW model (1989 US$).

Results from the empirical DDR literature are sensitive to the model estimated, the data series used to estimate the model, and how the data are smoothed and corrected for inflation. However, these models clearly make a difference: using the certainty-equivalent discount factors from Groom et al.'s (2007) preferred model, rather than a constant exponential discount rate, increases the social cost of carbon, based on Nordhaus damage estimates, by 250 percent (from $5.74 to $14.44).

The appropriateness of using these estimates for policy depends on one's view of what the discount rate represents. The empirical literature forecasts the risk-free rate of return on

[8] In the state-space model $r_t = \pi + a_t r_{t-1} + e_t$, where $a_t = \sum \lambda_i a_{t-1} + u_t$. e_t and u_t are serially independent, zero-mean, normally distributed random variables, whose distributions are uncorrelated. The author compares the models using the root mean squared error of within- and out-of-sample predictions.

capital, as measured by the yield on long-term government bonds. The literature on cost–benefit analysis dictates that the net benefits of a project in year t—in consumption units—are to be discounted to the present at the rate at which society would trade consumption in year t for consumption in the present. The consumption rate of discount may not equal the risk-free rate of return on capital. The next section presents arguments for a DDR following the consumption rate of discount approach.

III. Declining Discount Rates Based on the Ramsey Formula

In the context of intergenerational discounting, the consumption rate of discount is usually approached from the perspective of a social planner who wishes to maximize the social welfare of society (Dasgupta 2008; Goulder and Williams 2012).[9] In evaluating investment projects, a social planner would be indifferent between £1 received at time t and £ε today if the marginal utility of £ε today equaled the marginal utility of £1 at time t.[10]

$$u'(c_0)\varepsilon = e^{-\delta t}u'(c_t) \qquad (7)$$

Equation (7) assumes that the utility received from a given level of consumption is constant over time, but that future utility is discounted at the rate δ. Solving equation (7) for ε yields

$$\varepsilon = \frac{e^{-\delta \cdot t}u'(c_t)}{u'(c_0)} = e^{-\rho_t \cdot t} \qquad (8)$$

where ρ_t denotes the consumption rate of discount. If we assume that $u(c)$ exhibits constant relative risk aversion (CRRA), $u(c) = c^{(1-\eta)}/(1-\eta)$, then ρ_t can be written using the familiar Ramsey formula

$$\rho_t = \delta + \eta \cdot g_t \qquad (9)$$

[9] Dasgupta (2011) suggests an alternative approach in which an infinitely lived representative agent is replaced by dynasties of agents with finite lives. This model distinguishes between an individual's preferences for allocating consumption across his own lifetime from his preferences to bequeath resources to future generations.

[10] In this paper, c_t represents the average consumption of people alive at time t. In an intergenerational context, t is often interpreted as indexing different generations; however, it need not be. It can simply represent average consumption in different time periods, some of which may contain the same people.

where η is the coefficient of relative risk aversion (also the elasticity of marginal utility with respect to consumption) and g_t is the annualized growth rate of consumption between time 0 and time t.

In equation (9), δ is the rate at which society discounts the utility of future generations. A value of δ =0 says that we judge the utility of future generations to be equal to our utility, holding consumption constant. η describes (for any generation) how fast the marginal utility of consumption declines as consumption increases. Higher values of η imply that the marginal utility of consumption declines more rapidly as η increases. The standard interpretation of (9) is that the planner (society) will discount the utility of consumption of future generations at a higher rate because future generations are wealthier (i.e., the higher is the rate of growth in consumption, g_t). To illustrate, if g_t =1.3 percent per capita, consumption in 200 years will be 11 times higher than it is today. So it makes sense to discount the utility of an extra dollar of consumption received 200 years from now. And the planner will discount it at a higher rate the faster the marginal utility of consumption decreases as consumption rises.

I defer the discussion of how a planner should choose the parameters δ and η to the next section, and focus now on conditions under which (9) will yield a consumption discount rate that declines over time. Assuming that δ and η are constants and g_t is certain, a DDR requires the rate of growth in consumption to decline over time. The rate of growth in consumption is, however, uncertain—especially over long horizons.

If shocks to consumption are independently and identically normally distributed, uncertainty about g_t reduces ρ_t, but ρ_t will be constant. Suppose that $ln(c_t/c_0) = \sum_{i=1,t} ln(c_i/c_{i-1})$, where $ln(c_i/c_{i-1})$, the proportionate change in consumption at i, is independently and identically normally distributed with mean μ_g and variance σ_g^2. This leads to the extended Ramsey rule (Gollier 2007)[11]

$$\rho = \delta + \eta\mu_g - 0.5\, \eta^2\zeta_g^2 \qquad (10)$$

The last term in (10) is a precautionary effect: uncertainty about the rate of growth in consumption reduces the discount rate, causing the social planner to save more in the present.12

[11] This result goes back at least as far as Mankiw (1981).

[12] A necessary condition for this to hold is that the planner be prudent (i.e., that the third derivative of *u(c)* be positive), which is satisfied by the CRRA utility function.

The magnitude of the precautionary effect is, however, likely to be small, at least for the United States. Suppose that $\delta = 0$, and $\eta=2$, as suggested by Dasgupta (2008). Using annual data from 1889 to 1978 for the United States, Kocherlakota (1996) estimated μg to be 1.8 percent and ζg to equal 3.6 percent. This implies that the precautionary effect = 0.002592 and that ρ = 3.34 percent (rather than 3.6 percent, as implied by equation (8)).[13]

As equation (10) illustrates, independently and identically normally distributed shocks to consumption with known mean and variance result in a constant consumption rate of discount. The consumption rate of discount may decline if shocks to consumption are correlated over time, or if the rate of change in consumption is independently and identically distributed with unknown mean or variance.

Gollier (2007) proves that if shocks to consumption are positively correlated and $u(c)$ exhibits CRRA, ρ_t will decline.[14] The intuition behind this is that positive shocks to consumption make future consumption riskier, increasing the strength of the precautionary effect in equation (10) as t increases. A possible form that shocks to consumption could take is for $ln(c_t/c_{t-1}) \equiv x_t$, the percentage growth in consumption at t, to follow an AR(1) process

$$x_t = \varphi x_{t-1} + (1- \varphi)\mu + u_t \qquad (11)$$

where u_t is independently and identically normally distributed with constant variance. Mathematically, equation (11) will generate a DDR, provided $0 < \varphi < 1$. To be precise, the precautionary effect is multiplied by the factor $(1- \varphi)^{-2}$ as t goes to infinity (Gollier 2008).

Various models of per capita consumption growth have been estimated for the United States (e.g., Cochrane 1988; Cecchetti et al. 2000), and these could be used to empirically estimate a DDR using the extended Ramsey formula. However, the rate of change in per capita consumption in the United States is less persistent than the yield on the long-term bond reported in section II. This approach is therefore unlikely to yield a certainty-equivalent rate that declines as rapidly as shown in Figures 3 and 4. In the case of equation (11), Gollier (2008) reports an estimate of φ = 0.3, based on the literature, which implies a very gradual decline in the certainty-equivalent discount rate. The same is true of the certainty-equivalent discount rate based on the

[13] Gollier (2011) finds that the size of the precautionary effect is much larger for other countries, especially developing countries.

[14] Formally, Gollier shows that if $ln(c_i/c_{i-1})$ exhibits positive first-order stochastic dependence and $u'''(c) > 0$, ρ_t will decline as t increases.

regime-switching model of Cecchetti et al. (2000). The certainty-equivalent rate in the positive growth regime declines from 4.3 percent today to 3.4 percent after 100 years.

The approach to parameterizing the extended Ramsey formula described in the previous paragraphs is based on the assumption that the nature of the stochastic consumption-growth process can be adequately characterized by econometric models estimated using historical data. The consumption-based asset pricing literature suggests that this is not the case.[15] To quote Weitzman (2007,1102), ―People are acting in the aggregate like there is much more . . . . subjective variability about future growth rates than past observations seem to support.” This argues for treating μ_g and σ_g as uncertain. Subjective uncertainty about the trend and volatility in consumption growth, as modeled in Weitzman (2007, 2004) and Gollier (2007), will lead to a declining certainty-equivalent discount rate.

Weitzman (2004) considers the case in which x_t is independently and identically normally distributed with mean μ_g and variance σ_g^2. The planner is uncertain about μ_g and updates his diffuse prior distribution over μ_g using n observations on x_t. This leads to the following equation for the certainty-equivalent discount rate

$$\rho_t = \delta + \eta\ \mu_g - 0.5\ \eta^2\zeta_g^2 - 0.5\ \eta^2\zeta_g^2(t/n) \tag{12}$$

Bayesian updating adds a fourth term to the extended Ramsey rule—a *statistical forecasting effect*—which causes ρ_t to decline with t, conditional on n and σ_g^2. Intuitively, Bayesian learning generates a positive correlation in the perceived growth of consumption.[16]

The form of the planner‘s subjective uncertainty about the mean rate of growth in consumption clearly influences the path of the certainty-equivalent discount rate. The assumptions in Weitzman (2004) cause the certainty-equivalent discount rate to decline linearly, eventually becoming negative (see equation (12)). Gollier (2007, 2008) presents examples that yield nonnegative paths for the certainty-equivalent discount rate.

[15] The extended Ramsey formula does a poor job of explaining the equity premium puzzle: the large gap between the mean return on equities and risk-free assets.

[16] Weitzman (2007) also considers the case where σ_g^2 is unknown and is assumed to have an inverted Gamma distribution. In this case, Bayesian updating transforms the distribution of x_t from a normal into a Student t-distribution, which has fatter tails. The certainty-equivalent discount rate also declines to $-\infty$ in this case.

Gollier (2007) proves that, when the rate of growth in log consumption follows an RW and the mean rate of growth depends on θ [$\mu_g = \mu_g(\theta)$], the certainty-equivalent discount rate, R_t, is given by

$$R_t = \delta + \eta M_t \tag{13}$$

where M_t is defined by

$$exp(-\eta t\, M_t) = E_\theta\, exp\, [-\eta t(\mu_g(\theta) - 0.5\eta\sigma_g^2)] \tag{14}$$

As a result of Jensen's inequality, M_t (and R_t) will decline over time. Figure 5 demonstrates the path of R_t for the case of $\delta = 0$, $\eta = 2$, and $\sigma_g = 3.6$ percent. The mean rate of growth in consumption is assumed to equal 1 percent and 3 percent with equal probability. This yields a certainty-equivalent discount rate that declines from 3.8 percent today to 2 percent after 300 years—a path that closely resembles the French discounting schedule in Figure 1. The choice of other distributions for θ will, of course, lead to other DDR paths.

Uncertainty about the future rate of growth in per capita consumption can lead to a declining consumption rate of discount, assuming that shocks to consumption are positively correlated. Econometric models of per capita consumption growth can be used to parameterize the extended Ramsey formula, or subjective uncertainty about the trend and volatility of consumption growth can be used to derive a DDR. Econometric models of per capita growth in the gross domestic product (GDP) (Cochrane 1988; Cicchetti et al. 2000) are similar in form to the models used in the empirical ENPV literature. For the United States, however, these models do not suggest significant persistence in shocks to per capita consumption growth, implying that they will not generate a DDR that declines as rapidly as the DDRs in Figure 3 and 4.

Subjective uncertainty about the trend and volatility in consumption growth will also lead to a declining certainty-equivalent discount rate. The issue with this approach, from the perspective of policymakers, is how to use it to obtain a numerical DDR schedule. As the examples in this section illustrate, the shape of the DDR is very sensitive to distributional assumptions about μ_g and σ_g. And the fact that these distributional assumptions reflect subjective uncertainty makes it difficult to estimate them empirically.

IV. How To Parameterize the Extended Ramsey Formula

To parameterize the DDR using the consumption rate of discount requires estimates of δ and η as well as information about the process governing the growth of per capita consumption.

The Ramsey approach to discounting, which underlies the theory of cost–benefit analysis, is a normative approach. This implies that its parameters should reflect how society values consumption by individuals at different points in time; in other words, δ and η should reflect social values. The question is how these values should be measured.

δ and η as Ethical Parameters

Many people would agree with Frank Ramsey that it is ethically indefensible to discount the utility of future generations, except possibly to account for the fact that these generations may not exist. This implies that $\delta = 0$, or a number that reflects the probability that future generations will not be alive. Stern (2006), for example, assumes that the hazard rate of extinction of the human race is 0.1 percent per year.

The parameter η, which determines how fast the marginal utility of consumption declines as consumption increases, can be viewed as a measure of inequality aversion: it reflects the maximum sacrifice one generation should make to transfer income to another generation. To make this more concrete, Table 3 describes the maximum sacrifice that society believes a higher-income group (A) should make to transfer £1 to the lower-income group (B), as a function of η. When group A is twice as rich as group B and $\eta=1$, the maximum sacrifice is £2; when $\eta=2$, the maximum sacrifice is £4.

How, empirically, should η be determined? One approach is to examine the value of η implied by decisions that society makes to redistribute income, such as through progressive income taxes. Socially revealed inequality aversion in the United Kingdom, based on income tax schedules, is pictured in Figure 6. As the figure reveals, η has fluctuated considerably since the Second World War, with a mean of 1.6 (Groom 2011). It is also possible to elicit values of η and δ using stated preference methods. The issue here is whose preferences are to be examined and how. As Dasgupta (2008) has pointed out, it is important to examine the implications of the choice of η and δ for the fraction of output that a social planner would choose to save. Ceteris paribus, a lower value of η implies that society would choose to save a larger proportion of its output to increase the welfare of future generations.[17] The implications of the choice of δ and η would need to be made clear to the subjects queried.

[17] Dasgupta (2008) criticizes the 2006 Stern Review's choice of $\eta = 1$ because it leads to an absurdly high rate of saving along an optimal consumption path. Higher values of η lead to more reasonable ratios of saving to output.

In equations (8) and (9), η also represents the coefficient of relative risk aversion. A large literature infers empirical estimates of η from savings and investment decisions in financial markets; however, these estimates do not reflect intergenerational consumption tradeoffs and are inappropriate as estimates of η in a social welfare function. It would, however, be possible to choose the coefficient of relative risk aversion from a social perspective by confronting respondents with lotteries over consumption.

Should δ and η Be Based on Observed Behavior in Financial Markets?

The suggestion that η be estimated from observed behavior in financial markets raises the broader issue of whether the consumption rate of discount should reflect observed behavior and/or the opportunity cost of capital. The *descriptive* approach to social discounting (Arrow et al. 1996), epitomized by Nordhaus (1994, 2007), suggests that δ and η should be chosen so that ρ_t approximates market interest rates. In base runs of the Nordhaus DICE model, $\delta = 1.5$ and $\eta = 2$. DICE is an optimal growth model in which g_t and ρ_t are determined endogenously. ρ_t ranges from 6.5 percent in 2015 to 4.5 percent in 2095 (Nordhaus 2007).

This raises the question: should we expect the consumption rate of discount in equation (8) to equal the rate of return to capital in financial markets, and, if not, what should we do about this? From a social welfare perspective, the consumption rate of discount in (8) will equal the marginal product of capital along an optimal consumption path. If, for example, the social planner chooses the path of society's consumption in a one-sector growth mode, ρ_t will equal the marginal product of capital along an optimal path. What if society is not on an optimal consumption path? Then theory (Dasgupta et al. 1972, Dasgupta 2008) tells us that we need to calculate the social opportunity cost of capital—that is, we need to evaluate the present discounted value of consumption that a unit of capital displaces—and use it to value the capital used in a project when we conduct a cost–benefit analysis. But once this is done—once all quantities have been converted to consumption equivalents—the appropriate discount rate to judge whether a project increases social welfare is ρ_t.

V. Should Governments Use a DDR in Cost–Benefit Analyses?

In benefit–cost analysis, the net benefits of a project in year t (in consumption units) are to be discounted to the present at the rate at which society would trade consumption in year t for consumption in the present. If utility of consumption exhibits CRRA, the consumption rate of discount (ρ_t) is given by the Ramsey formula, $\rho_t = \delta + \eta g_t$, where δ is the utility rate of discount, η *is* the elasticity of marginal utility of consumption, and g_t is the average rate of growth in consumption between t and the present. If uncertainty about the rate of growth in consumption is persistent, the certainty-equivalent consumption rate of discount will decline as t increases. This uncertainty in future consumption growth rates may be estimated econometrically based on historic observations; alternatively, it can be derived from subjective uncertainty about the mean rate of growth in mean consumption or its volatility.

The path from theory to a numerical schedule of the certainty-equivalent consumption rate of discount is, however, difficult. It requires estimates of δ, η and assumptions about the process generating g_t. These are all difficult to estimate and to defend (or to explain!) to regulators. This suggests that a second-best approach may be called for.

The ENPV approach is less theoretically elegant and does not measure the consumption rate of discount as given by the Ramsey formula. It is, however, empirically tractable and corresponds to the approach currently recommended by OMB (2003) for discounting net benefits when expressed in consumption units. OMB (2003,11) acknowledges that ―the effects of regulation do not always fall exclusively or primarily on the allocation of capital. When regulation primarily and directly affects private consumption . . . a lower discount rate is appropriate." A discount rate of 3 percent is meant to represent this possibility and is approximated by the real rate of return on long-term government debt.[18] The empirical ENPV literature has focused on models of the rate of return on long-term government debt. And, in the United States, the literature suggests that the certainty-equivalent rate is declining over time.

For the DDR to be implemented empirically would require reanalysis of the choice of OMB's consumption rate of discount. Use of a DDR schedule would, however, avoid problems that arose in a recent benefit–cost analysis of fuel economy standards in the United States. The social cost of capital was calculated using a discount rate of 2.5 percent, whereas the fuel savings associated with the rule were discounted at 3 percent. Different categories of benefits received in the same year were discounted at different rates, leading to inconsistency in the calculation of benefits. This problem could be avoided by using a DDR schedule.[19]

[18] Interestingly, OMB (2003,11) indicates that this rate is sometimes referred to as the *social rate of time preference* and ―simply means the rate at which ‗society' discounts future consumption flows to their present value."

[19] It could, of course, also be avoided by using the same constant discount rate (e.g., 3 percent) to discount both sets of benefits.

In closing, I discuss whether time consistency may be an issue in using a DDR for project analysis. It is well known that the decisions of a planner who discounts the utility of future consumption at a constant exponential rate will make time-consistent decisions even though the consumption rate of discount may decline over time (Gollier et al. 2008). The situation is somewhat different in the case of an analyst who is faced with an uncertain discount rate in an ENPV context. If an analyst were to evaluate future net benefits using the discounting schedule in Gamma Discounting (Weitzman 2001) in 2012, then, if the schedule did not change over time, a program that passed the benefit–cost test in 2012 would not necessarily pass it in 2022, depending on the time pattern of net benefits.

Of course, if new information becomes available that alters the DDR schedule, the analyst will want to reevaluate the ENPV of the program. Because new information is available, a reversal of the outcome of the cost-benefit analysis would not constitute time inconsistency. Newell and Pizer (2003) argue that an analyst, when using historical data to estimate a DDR, will naturally update estimates of the DDR as more information becomes available. This obviates the problem of time inconsistency. In a regulatory setting, however, such updating may occur only infrequently.[20] This is an issue deserving more thought.

[20] The U.K. discount rate schedule in Figure 1 has been in place since 2003 (H.M. Treasury 2003).

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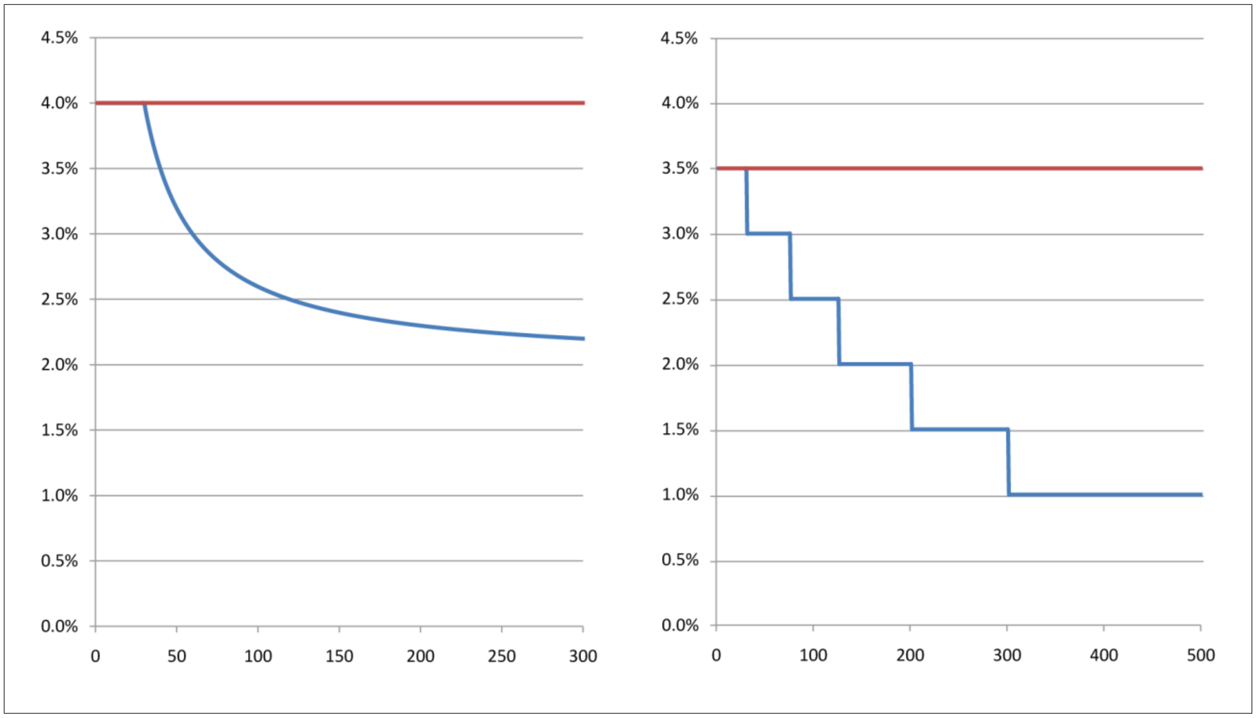
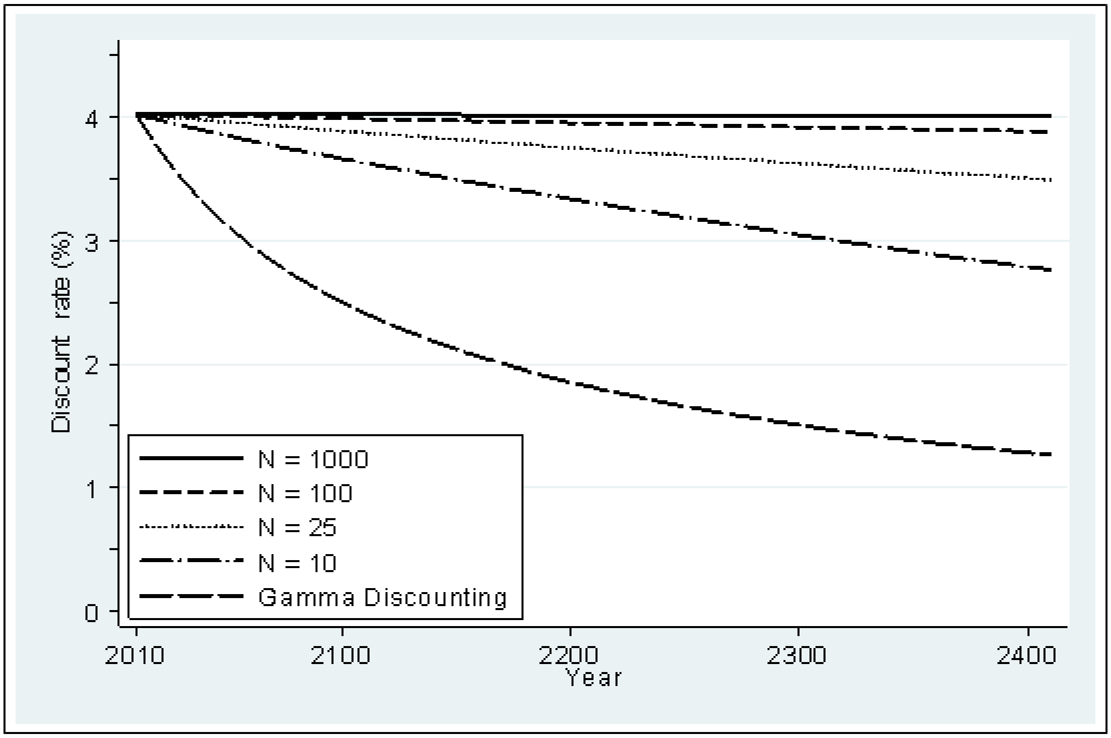
Figures and Tables

Figure 1. Declining Discount Rates in France and the United Kingdom



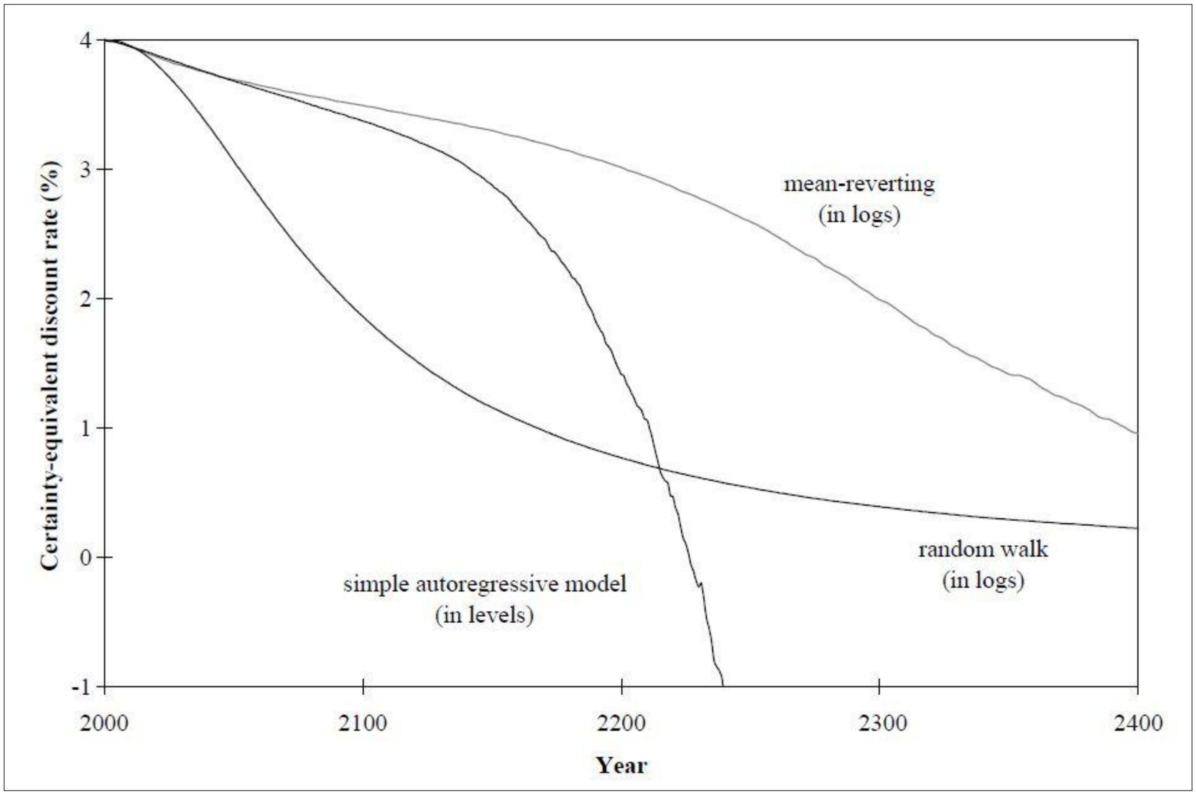
Source: Sterner (2011)

Figure 2. Certainty-Equivalent Discount Rates: Freeman–Groom vs. Weitzman



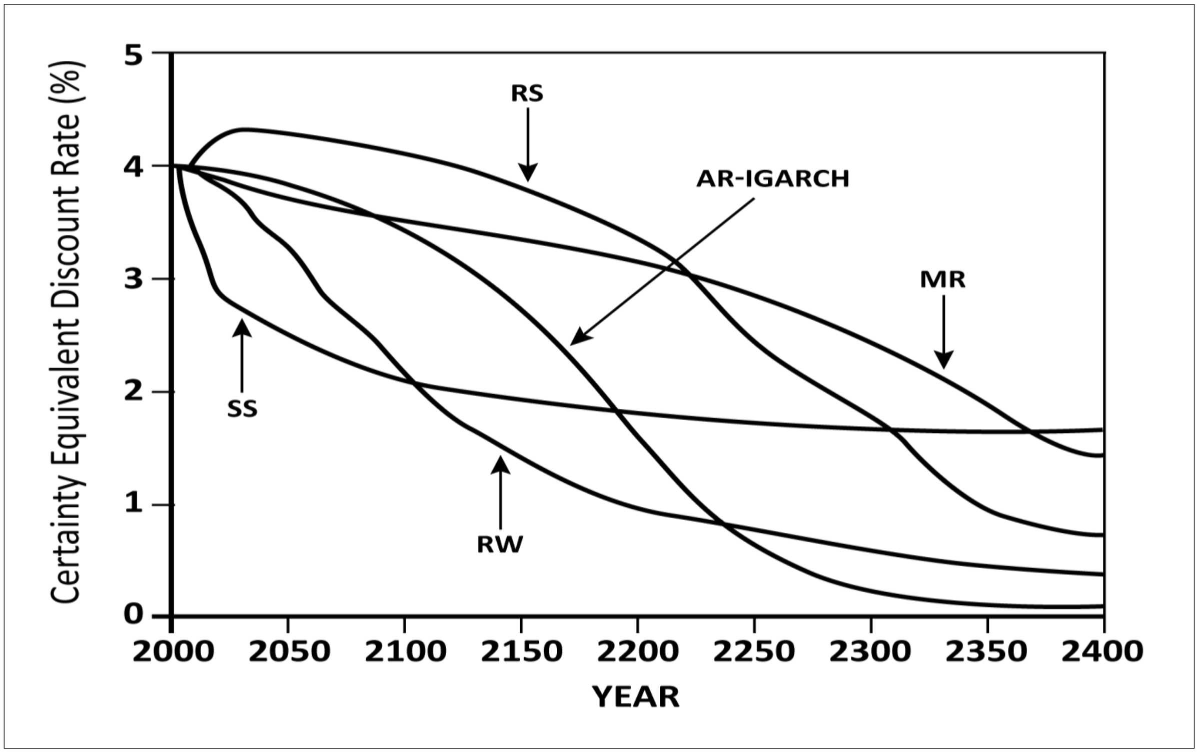
Source: Groom (2011)

Figure 3. Forecasts of Certainty-Equivalent Discount Rates from Newell and Pizer (2003)



Source: Newell and Pizer (2003)

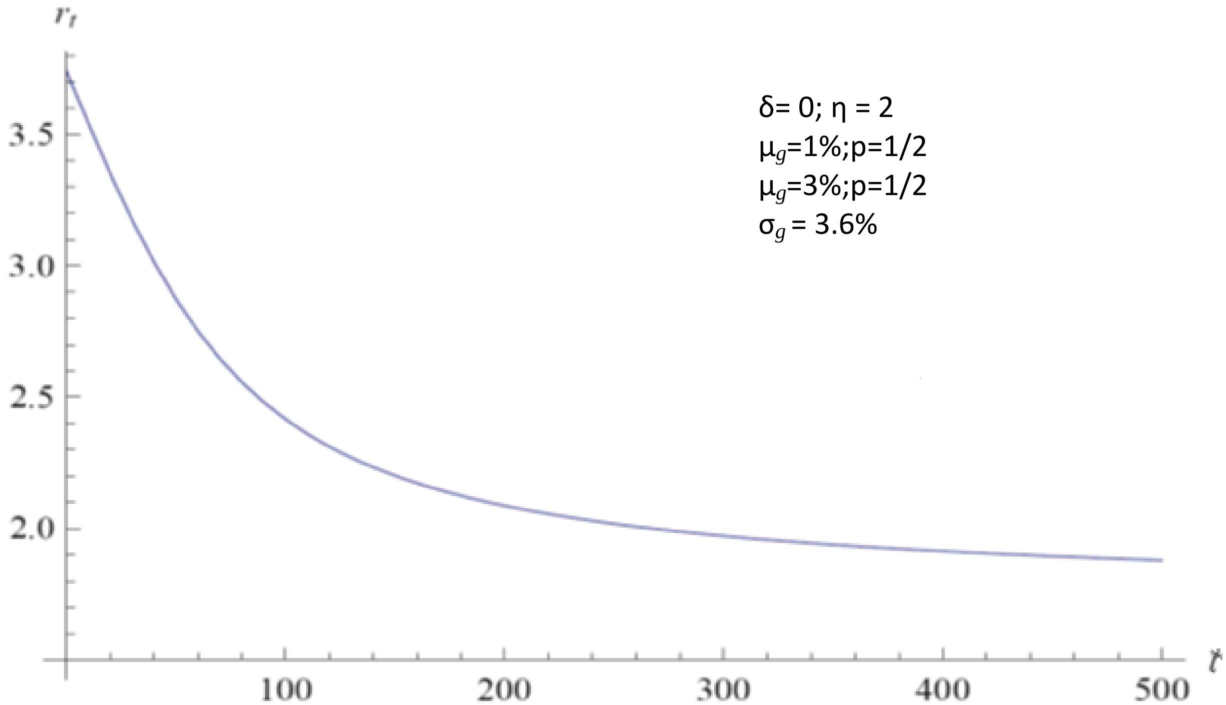
Figure 4. Forecasts of Certainty-Equivalent Discount Rates from Groom et al. (2007)



Notes: RS = regime-switching, MR = mean-reverting, SS=state-space, RW=random walk, AR-IGARCH= autoregressive integrated generalized autoregressive conditional heteroskedasticity.

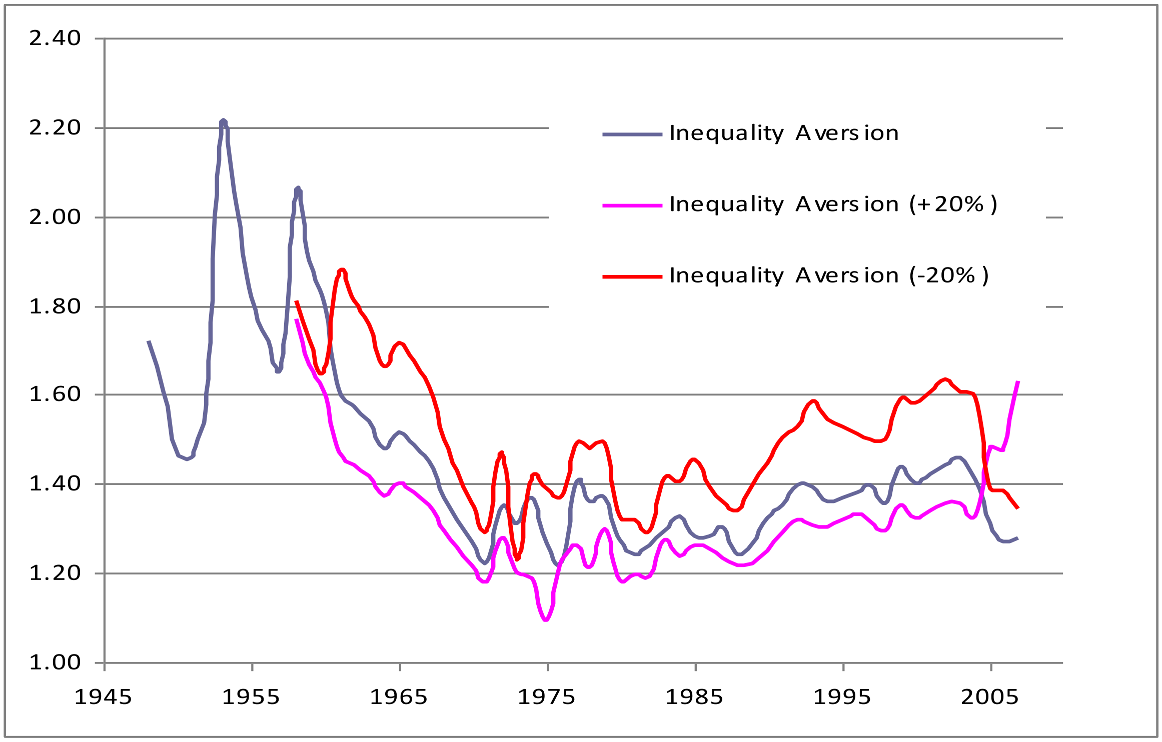
Source: Groom et al. (2007)

Figure 5. Certainty-Equivalent Discount Rate Assuming per Capita Consumption Follows a Random Walk with Uncertain Mean, $\mu = \mu(\theta)$



Source: Gollier (2008)

Figure 6. Estimates of Inequality Aversion (η) Based on the U.K. Income Tax



Source: Groom (2011)

Table 1. Present Value of a Cash Flow of £1,000 Received after t Years

t	Scenario A: 4%	Scenario B: 1% or 7%	Certainty-equivalent discount rate (R_t)
1	960.7894	961.2218	0.0394
10	670.3200	700.7114	0.0313
50	135.3353	318.3640	0.0128
100	18.3156	184.3957	0.0102
150	2.4788	111.5788	0.0101
200	0.3355	67.6681	0.0101
300	0.0061	24.8935	0.0101
400	0.0001	9.1578	0.0101

Source: Gollier et al. (2008)

Table 2. Discount Rate Schedule from Weitzman (2001)

Time period	Name	Marginal discount rate (Percent)
Within years 1 to 5 hence	*Immediate* Future	4
Within years 6 to 25 hence	*Near* Future	3
Within years 26 to 75 hence	*Medium* Future	2
Within years 76 to 300 hence	*Distant* Future	1
Within years more than 300 hence	*Far-Distant* Future	0

Source: Weitzman (2001)

Table 3. Maximum Acceptable Sacrifice from Group A To Increase Income of Group B by £1

η	Group A income = 2*Group B income	Group A income = 10*Group B income
0	1.00	1.00
0.5	1.41	3.16
1	2.00	10.00
1.5	2.83	31.62
2	4.00	100.00
4	16.00	10000.00