Refresher:

Foundations of Decision Theory

Roger Cooke
Chauncey Starr Senior Fellow
Resosurces for the Future
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- **Decision theory**
- **Uncertainty**
- **Preference**

Uncertainty is that which disappears when we become certain

Decision Theory

L.J. Savage 1954:

- Choice behavior of a ‘rational individual’ can be represented as expected utility with
  - Unique probability measure
  - Unique Utility up to ‘0’and ‘1’.
- BUT groups are not rational individuals...aim for ***rational consensus***

Paradox of Majority Preference

1/3 prefer Mozart > Hayden > Bach
1/3 prefer Hayden > Bach > Mozart
1/3 prefer Bach > Mozart > Hayden

THEN 2/3's prefer

Bach > Mozart
Mozart > Hayden
Hayden > Bach

Distinguish and Separate

- Uncertainty wrt ‘what is the case’
- Values, utilities, preferences

Who does what?

Uncertainty $\Rightarrow$ Domain experts

Utility, preferences $\Rightarrow$ Policy makers, stakeholders, voters

Uncertainty

Why represent uncertainty as probability??

Foundations:
Axioms + Interpretation

Interpretation: aka
- **operational definitions**
- epistemic rules
- rules of correspondence
- etc etc

Operational Definitions

- The philosophy of science: semantic analysis: Mach, Hertz, Einstein, Bohr

- A Modern rendering:

IF BOB says

"The Loch Ness monster exists with degree of possibility 0.0731"

to which sentences in the natural language
not containing "degree of possibility"
is BOB committed?

Operational definition: Subjective probability

Consider two events:

F: France wins next World Cup Soccer tournament

U: USA wins next World Cup Soccer tournament.

Two lottery tickets:

L(F): worth $10,000 if F, worth $100 otherwise

L(U): worth $10.000 if U, worth $100 otherwise.

John may choose ONE .

John's degree belief (F) >= John's degree belief (U)

is operationalized as

John chooses L(F) in the above choice situation

Ask yourself:

B: Belgium wins next World Cup Soccer tournament.

$L(F) > L(U)$; $L(U) > L(B)$; $\Rightarrow$ $L(F) > L(B)$??

$L(F) > L(U) \quad \Rightarrow \quad L(F \text{ or } B) > L(U \text{ or } B)$??

(plus some technical axioms)

Then there is a UNIQUE probability P which represents degree of belief (Savage's representation theorem):

$DegBel(F) > DegBel(U) \Leftrightarrow P(F) > P(U)$

This can be measured observing INDIVIDUAL choice behavior

Operational definition, frequentist probability

“Prob(A) = 0.873” means

“The relative frequency of occurence of A in a REFERENCE CLASS is 0.873.”

Reference class must be a “collective”, a sequence of ‘trials’ for A, such that every ‘decidable subsequence’ has same limit rel. frequency. (von Mises, Kolmogorov)

MUST SPECIFY REF. CLASS

To clarify

- You can be uncertain about a limit rel. frequency
- You can learn about a rel. freq. thereby reducing your uncertainty
- You can quantify your uncertainty conditional on, say, X, and be uncertain about X
- You cannot be uncertain about your uncertainty in any other useful sense.

  "my uncertainty in success is 0.7, but my uncertainty in my uncertainty is 0.5, and my uncertainty in my uncertainty of my uncertainty is 0.3...." DON'T GO THERE

Other interpretations of Probability axioms

- Classical interpretation (Laplace) ‘ratio of favorable cases to all equi-possible cases’

- Logical Interpretation (Keynes, Carnap) ‘partial logical entailment’

Neither were able to provide successful operational definitions.

Alternative representations of uncertainty have no foundation

Fuzzy sets: many axiomatizations, no operational definitions

Degree of Possibility: no operational definitions

(see however *Eur. J. of Oper. Res. 128, 459-478.p 477).*

EG if $\mu_A(x)$ = fuzzy membership of x in A

DOES

$\mu_{winworldcup}(FRANCE) > \mu_{winworldcup}(USA)$

ENTAIL

L(F) > L(U) ????

Does it entail anything not involving “fuzzy”?

CAN fuzziness represent uncertainty?

$\mu_{man}(Quincy) = \mu_{woman}(Quincy) = ½$

$\Rightarrow$

$\mu_{man\ AND\ woman}(Quincy) =$

$\text{Min}\ \{\mu_{man}(Quincy), \mu_{woman}(Quincy)\} = ½$

L(Quincy is a man) ~ L(Quincy is a hermaphrodite) ??

EJ for RATIONAL CONSENSUS:

Parties pre-commit to a method which satisfies necessary conditions for scientific method:

- ***Traceability/accountability***
- ***Neutrality (don't encourage untruthfulness)***
- ***Fairness (ab initio, all experts equal)***
- ***Empirical control (performance meas't)***

Withdrawal post hoc incurs burden of proof.

What is a GOOD subjective probability assessor?

- Calibration, statistical likelihood
  - Are the expert's probability statements statistically accurate?
- Informativeness
  - Probability mass concentrated in a small region, relative to background measure
- Nominal values near truth
- ?

Expert elicitation techniques

- Delphi
- Nominal group techniques
- Group nomination
- Team building, decision conferencing, etc

Key question: How do we measure performance

Credibility via performance, period.

Preference:

Stakeholder Preference Theory

Simple paired comparisons (consumer research)

MCDM

MAUT

AHP

Etc etc

Consumer sampling $\Rightarrow$ valuation of competing products

Ask random consumer:

Do you prefer

APPLES to BANANAS?
APPLES TO ORANGES
BANANAS TO ORANGES?
ETC

Find a value function V(x) that represents preferences

Paired Comparison Data Analysis

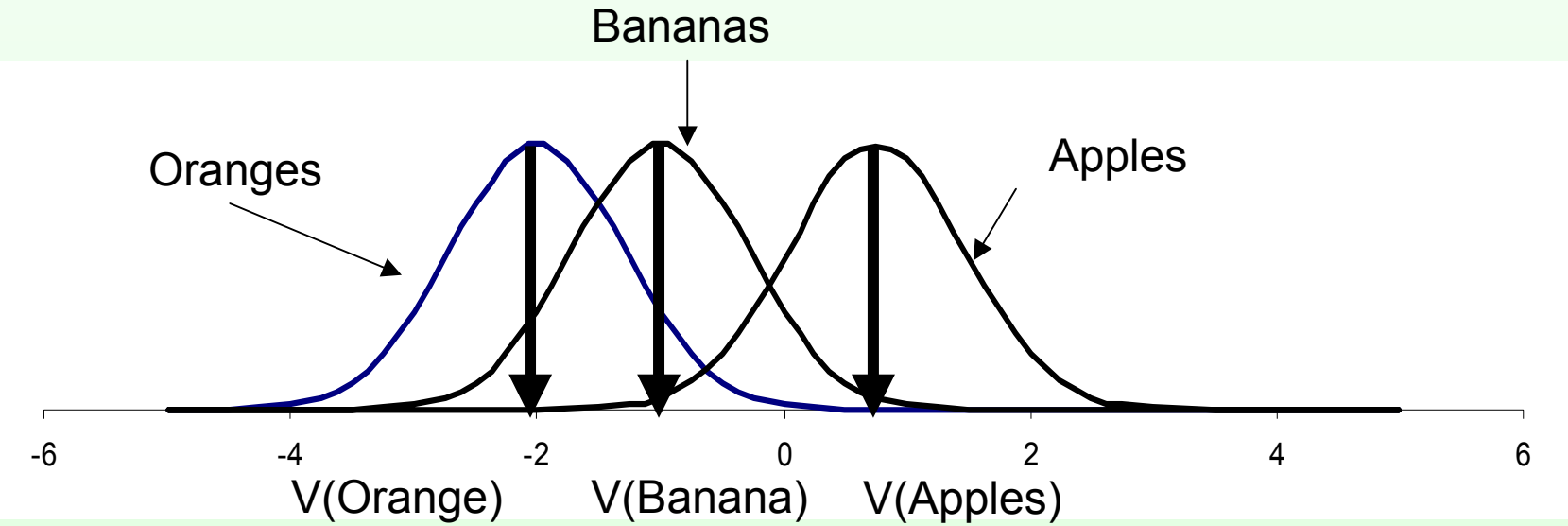
- Is each customer's preference non-random?

- Is the agreement between customers non-random?
  - Coefficient of agreement
  - Coefficient of concordance

Thurstone's model

Assume values are normally distributed over stakeholders:

Find relative placement that reproduces preferences



Values are determined up to "0 and 1".

Bradley-Terry

Assume that pairwise preference is related to value V(x) as:

$$\%(APPLES > BANANAS) = \frac{V(APPLES)}{V(APPLES)+V(BANANAS)}$$

$$= PROB\ (\ apples \mid apples\ \ or\ bananas\ )$$

Values determined up to a constant

Hands-on

- Expert elicitation
  - Air quality or
  - SARS
- Stakeholder preference: 6 fuel policies