Employment Dynamics in General Equilibrium: Model Documentation

Marc Hafstead, Roberton Williams III, Luis Fernandez Intriago

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About the Author

Marc Hafstead is a Fellow at Resources for the Future (RFF). He joined RFF in 2013 from Stanford University. He is also the director of the Carbon Pricing Initiative and Climate Finance and Financial Risk Initiative at RFF.
Roberton C. Williams III is a Senior Fellow at RFF. In addition to his position at RFF, he is a professor at the University of Maryland, College Park and a research associate at the National Bureau of Economic Research (NBER).
Luis Fernandez Intriago is a Postdoctoral Fellow at RFF. He joined RFF after receiving his PhD in economics at Arizona State University.

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Overview

The Employment Dynamics in General Equilibrium (EDGE) model is an extension of modeling work published in Hafstead and Williams III (2018) and Hafstead et al. (2022). EDGE introduces search frictions (Mortensen and Pissarides (1994)) into multi-sector general equilibrium models of the US economy.[1] The search friction creates processes of job creation and job destruction, leading to flows of workers into and out of unemployment each period. Unlike full-employment models (total hours demand must equal total hours supply), unemployment is an equilibrium concept in search models.

Both Hafstead and Williams III (2018) and Hafstead et al. (2022) used EDGE to evaluate how environmental policies impact unemployment and the reallocation of workers across sectors. However, the EDGE model is not an environmental model per se and could be used to evaluate non-environmental policy-driven employment dynamics.

The benchmark model is a simple two-sector extension of Shimer (2010) with variable hours. The model features a closed economy with an emissions abatement function for each sector with labor as the only input into production. The benchmark EDGE model is extended to include multiple sectors and international trade using a Social Accounting Matrix framework for production, on-the-job search, job promotions, and capital.[2]

In each model, recruiters and job searchers are paired through a matching process. The number of matches is a function of recruiting effort and the number of people looking for jobs. After a match occurs, the new worker and the firm bargain over wages and the worker begins working the following period.[3] At the end of each period, a fraction of workers loses their job (or quit in the on-the-job search model).

Environmental policies disrupt the equilibrium employment dynamics in both the short and long run. As shown in Hafstead and Williams III (2018), the introduction of environmental policies leads to a reallocation of workers across sectors and a (small) increase in equilibrium unemployment.

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[1] In Hafstead et al. (2022), EDGE was referred to as a "Search-CGE" model.

[2] The documentation in this paper draws heavily on the model description sections from Hafstead and Williams III (2018) and Hafstead et al. (2022), and many portions of text in this documentation are copied verbatim from those papers.

[3] Typically, firms and its existing workers also renegotiate wages each period. The staggered wage assumption limits this re-bargaining process.

1. Benchmark Model

The benchmark model of EDGE is based on the model in Hafstead and Williams III (2018). It is referred to as EDGE-BENCHMARK.

1.1. Matching

Without loss of generality, the measure of workers is normalized to one. All workers are ex-ante identical - there is no heterogeneity in productivity or disutility of work across workers and sectors - but job finding probabilities are allowed to be a function of the industry the worker previously worked in.

Let n_j denote the measure of workers in sector j. Aggregate employment is given by $\bar{n} = \sum_j n_j$; the unemployment rate (and the measure of unemployed workers) is given by $\bar{u} = 1 - \bar{n} = \sum_i u_i$, where u_i is the number of unemployed workers previously employed in the sector i. Unemployed workers search indiscriminately across sectors, but the probability of a match depends on the unemployed worker's most recent employment history. Let v_j denote the number of recruiters in each sector, and let h_j denote the number of hours worked by the recruiter in a period.

The EDGE model's generalized matching function is given by

$$m_{ij} = \mu_j v_j h_j u_i \left[\xi \left(\sum_k v_k h_k \right)^{-\gamma_j} \left(\sum_k u_k \right)^{\gamma_j - 1} + (1-\xi)(v_j h_j)^{-\gamma_j} (u_i)^{\gamma_j - 1} \delta_{ij} \right] \quad (1)$$

where μ_j and γ_j are the matching efficiency and matching elasticity parameters, respectively, ξ is a parameter that controls for the degree of matching across sectors (for $\xi = 0$, a worker last employed in industry i can only match to industry i, for $\xi = 1$, matching is independent of the industry in which a worker was last employed, and in between, the number of cross-industry matches is proportional to ξ), and δ_{ij} is the Kronecker delta (equal to one if $i = j$ and zero otherwise). This generalized matching function with cross-industry switching frictions has a number of key properties.

- For $\xi = 0$, this reduces to a Cobb-Douglas matching function in each industry, with no cross-industry matches
- For $\xi = 1$, this reduces to the multi-sector version of the constant-returns-to-scale matching function used by Shimer (2010).

- Holding all other variables constant, the number of cross-industry matches is proportional to ξ.
- When μ_j, γ_j, and $v_j h_j / u_j$ are each constant across j, the total matches to each industry $\sum_i m_{ij}$ and total matches from each pool of unemployed workers $\sum_j m_{ij}$ each stay constant as ξ varies between 0 and 1.

To define the job finding probabilities and recruiting productivity, there are three measures of labor market tightness - $\theta_i j = \frac{v_j h_j}{u_i}$ is the ratio of recruitment effort in sector j to unemployed workers from sector i, $\theta_j = \frac{v_j h_j}{\bar{u}}$ is the ratio of recruitment effort in sector j to the total number of unemployed workers, and $\bar{\theta} = \sum_j \theta_j$ is the ratio of total recruitment effort to the total number of unemployed workers. As the recruitment effort increases in any given sector or there are less unemployed workers, the labor market becomes more tight - it becomes easier to find jobs but harder to recruit for workers; when recruitment effort declines or there are more unemployed workers, the labor market eases - it becomes harder to find jobs but easier to recruit workers.

Recruiting productivity q_j is defined as the number of matches per recruitment effort, $q_j = \sum_i m_i j / (v_j h_j)$; the job finding probability ϕ_{ij} is defined as the number of matches per unemployed worker, $\phi_{ij} = m_{ij}/u_i$. Using equation (1), both recruiting productivity and job finding probabilities as a function of labor market tightness,

$$q_j = \mu_j \left[\xi \bar{\theta}^{\gamma_j} + (1-\xi)\theta_{ii}^{-\gamma_j} \right] \tag{2}$$

$$\phi_{ij} = \mu_j \left[\xi \theta_j \bar{\theta}^{-\gamma_j} + (1-\xi)\delta_{ij}\theta_{ij}^{1-\gamma_j} \right] \tag{3}$$

1.2. Households

We use a representative household framework and assume full insurance within the household. This assumption implies that the marginal utility of consumption is equalized across workers and is independent of both past and current employment status.[4] Employed workers in sector j receive an hourly wage w_j and work h_j hours. Given labor income taxes at the rate τ_L , after-tax earnings per worker in sector j are given by $(1-\tau_L)w_j h_j$. Unemployed workers work zero hours and receive unemployment compensation b, which is held constant in real terms over time. Workers in each sector become unemployed at a fixed exogenous probability π_j each period.[5]

[4] This common assumption within the search literature, due originally to Merz (1995), greatly simplifies the problem of the household.

[5] In the benchmark EDGE model without on-the-job search, π_j represents all forms of job separations - layoffs, quits, and retirements.

The household owns the firms and has access to a complete set of state-contingent claims. Let Q denote the price of an Arrow security that delivers one unit of consumption the following period. The household also receives lump-sum transfers T from the government (taxes if negative).

Members of the household gain utility from consumption and disutility from work. The period felicity function is

$$U(\bar{C}, h) = \frac{C^{1-\sigma}}{1-\sigma} - \psi \frac{h^{1+\chi}}{1+\chi} \tag{4}$$

where σ represents the intertemporal elasticity of consumption, ψ represents a disutility from work parameter and χ is the Frisch elasticity of labor supply. Given that preferences are separable in consumption and hours worked, consumption will be independent of the employment status of the worker. Let $\bar{p}$ denote the price of the aggregate consumption good.

The household discounts future utility with a discount factor β. Households take the job finding rates ϕ_{ij} as given. The problem of the household is to maximize lifetime-discounted utility. This problem can be written dynamically as a Bellman equation, where the state of the household is given by the value of its current assets, B, and the distribution of workers across sectors, n_j, and the distribution of unemployed workers across their previous sector of employment u_i,

$$V(B, n_j, u_i) = \max_{\bar{C}, B} \left[\sum_j n_j U(\bar{C}, h_j) + \sum_i u_i U(\bar{C}, 0) + \beta E \left[V(B', n_j', u_i' \right] \right] \tag{5}$$

subject to the budget constraint,

$$\bar{p}\bar{C} + QB' \leq \sum_j (1-\tau_L) n_j w_j h_j + \sum_i u_i \bar{p} b + B + T \tag{6}$$

and the laws of motion for employment and unemployment by sector

$$\begin{aligned} n_j' &= (1-\pi_j) n_j + \sum_i u_i \phi_{ij} \\ u_i' &= \pi_i n_i + u_i (1 - \sum_j \phi_{ij}) \end{aligned} \tag{7}$$

The consumption good is created by aggregating the output of the two sectors according to a constant elasticity of substitution (CES) aggregation function. The price of the consumption good is

$$\bar{p} = (\gamma^c)^{\frac{\sigma^c}{1-\sigma^c}} \left[\sum_j (\alpha_j^c)^{\sigma^c} p_j^{1-\sigma^c} \right]^{\frac{\sigma^c}{1-\sigma^c}} \tag{8}$$

where α_j^c is the CES share parameter for good j, γ^c is the CES scaling parameter, and σ^c is the elasticity of substitution between goods produced by each sector. The amount of consumption demanded for goods produced by each sector is

$$c_j = (\gamma^c \alpha_j^c)^{\sigma^c} \left[\frac{p_j}{\bar{p}}\right]^{-\sigma^c} \bar{C} \tag{9}$$

The first-order condition for consumption is

$$\bar{p}\lambda = \sum_j n_j \bar{C}^{-\sigma} + \sum_i u_i \bar{C}^{-\sigma} = \bar{C}^{-\sigma} \tag{10}$$

where λ is the Lagrange multiplier for the budget constraint. By combining the first-order condition for next period's assets with the envelope condition with respect to current period assets, the Euler equation can be written as

$$Q = \beta \frac{\lambda'}{\lambda} \tag{11}$$

The marginal value of having a worker employed in a given sector is given by the envelope condition with respect to the number of workers employed in that sector and the marginal value of an unemployed worker with experience in a given sector is given by the envelope condition with respect to the number of unemployed workers from that sector

$$\begin{aligned} V_{n_j} =& U(\bar{C}, h_j) + \lambda(1-\tau_L) w_j h_j + (1-\pi_j)\beta V_{n_j}^{'} + \pi_j \beta V_{u_j}^{'} \\ V_{u_j} =& U(\bar{C}, 0) + \lambda \bar{p} b + \beta V_{u_j}^{'} + \beta \sum_k \phi_{jk} (V_{n_k}^{'} - V_{u_j}^{'}) \end{aligned} \tag{12}$$

1.3. Firms

The basic problem of the firm is to choose recruitment and abatement, conditional on its stock of workers, to maximize the value of the firm over time. Firms in each sector produce a sector-specific good using only a labor input, $y_j = A_j h_j l_j$, where l_j denotes the number of workers using a production technology that produces A_j units of output per hour worked. Given a measure of total workers n_j in the beginning of the period, the firm must decide to allocate workers to production, l_j, or recruitment, v_j, such that $l_j + v_j = n_j$.

Pollution emissions from production are given by

$$e_j = (1-\nu_j)\mu_j^e \bar{y}_j \tag{13}$$

where unconstrained emissions are a fixed multiple μ_j^e of net output $\bar{y}_j$ and ν_j is the fraction of unconstrained emissions that are abated. Net output is equal to gross output minus abatement costs, $\bar{y}_j = y_j - z_j$. The cost of abatement is modeled as a per (gross) unit of output cost $\bar{z}$ such that

$$z_j = \bar{z}(\nu_j) y_j \tag{14}$$

The domain of the abatement function $\bar{z}$ is $[0, 1]$. We assume that $\bar{z}(0) = 0$, $\lim_{x \longrightarrow 1} \bar{z}(x) = \infty$, $\bar{z}^{'}(0) = 0$ and $\bar{z}^{''}(0) > 0$. Firms must pay an emissions tax τ^e for all emissions that are not abated.

Firms are owned by households and discount future-period profits with the financial discount factor Q from the household problem. Following Shimer (2010), we define the recruiter ratio $\bar{v}_j = v_j / n_j$. Net output can be written as a function of total workers, the recruiter ratio, and abatement, $\bar{y}_j = A_j h_j n_j (1 - \bar{v}_j)(1 - \bar{z}(\nu_j))$. Firms take the endogenous recruiting productivity q_j as given when making recruiter decisions. The dynamic problem of the firm is to choose the recruiter ratio and abatement to maximize the value of the firm.

The Bellman equation is

$$J(n_j) = \max_{\bar{v}_j, \nu_j} \left[p_j^n(\nu_j) A_j h_j n_j (1 - \bar{v}_j)(1 - \bar{z}(\nu_j)) - (1 + \tau_p) n_j h_j w_j + E\left[Q J(n_j^{'}) \right] \right] \tag{15}$$

subject to

$$n_j^{'} = (1 - \pi_j) n_j + q_j \bar{v}_j h_j n_j \tag{16}$$

where p_j^n denotes the net price received by sector j for its output and τ_p denotes the payroll tax. The net price of output is equal to the price of the good minus the emissions tax payments, $p_j^n(\nu_j) = p_j - \tau^e \mu_j^e (1 - \nu_j)$.

The first-order condition with respect to the recruiter ratio sets the number of recruiters such that the marginal cost of switching an additional worker from production to recruitment equals the expected value of recruitment,

$$p_j^n(\nu_j) A_j (1 - \bar{z}(\nu_j)) = q_j E\left[Q J_{n_j}^{'} \right] \tag{17}$$

where $J_{n_j}^{'}$ is the value (to the firm) of an additional worker in the following period. The value of an additional worker can be derived from the envelope condition with respect to the number of workers and substituting out the first-order condition with respect to recruitment,

$$J_{n_j} = p_j^n(\nu_j) A_j h_j (1 - \bar{z}(\nu_j)) - (1 + \tau_p) h_j w_j + (1 - \pi_j) E\left[Q J_{n_j}^{'} \right] \tag{18}$$

This marginal value of a worker is equal to the marginal revenue minus compensation plus the expected continuation value times the probability that the match does not exogenously dissolve at the end of the period.

The first-order condition with respect to abatement will equate the marginal cost of abatement to the marginal benefit of abatement (i.e., lower emissions tax payments) such that

$$p_j^n(\nu_j)\bar{z}^{'}(\nu_j) = \tau^e \mu_j^e (1 - \bar{z}(\nu_j)) \tag{19}$$

1.4. Wage Bargaining

The model assumes that wages and hours are set through a Nash bargaining process, following the standard search literature. Let η denote the bargaining power of the employer. The equilibrium wages and hours are the solution to

$$\max_{w_j, h_j} J_{n_j}^{\eta} [V_{n_j} - V_{u_j}]^{1-\eta} \tag{20}$$

Nash bargaining implies that hours per worker are set to maximize the value of the match surplus. This means that the marginal value of an additional hour of work is equal to the disutility of an additional hour of work,

$$(1 + \tau_p)\psi h_j^{\chi} = (1 - \tau_L)\lambda p^n(\nu_j) A_j (1 - \bar{z}(\nu_j)) \tag{21}$$

Nash bargaining also implies that the match surplus is divided according to a constant share rule. The equilibrium after-tax pay packet for a worker is

$$\begin{aligned}(1 - \tau_L) h_j w_j = & \frac{1 - \tau_L}{1 + \tau_p}(1 - \eta)\left[p^n(\nu_j) A_j h_j (1 - \bar{z}(\nu_j))\right] \\ & \eta \left[\frac{\psi}{\lambda}\frac{h^{1+\chi}}{1+\chi} + \bar{p}b + \frac{\bar{\phi}_j}{\lambda}\right]\end{aligned} \tag{22}$$

where $\bar{\phi}_j = \beta \sum_k \phi_{jk}(V_{n_k}^{'} - V_{u_j}^{'})$. If wages are fully flexible, the equation above determines wages each period as wages are renegotiated each period.[6]

The model also allows for staggered wage bargaining as in Gertler and Trigari (2009). Staggered wage bargaining assumes that only a subset of firms within a sector can renegotiate wages each period. There exists a continuum of identical firms within each sector. Each period, a fraction $1 - \rho_j$ firms may renegotiate its contracts with its workers. The adjustment probability is independent over time, and although the length of contract is indeterminate, the average contract will last $1/(1 - \rho_j)$ periods. We follow Gertler and Trigari (2009) and assume that all workers at a given firm receive the same wage. Given the Poisson adjustment and common-within-firm wage assumption, we can write the average wage in each sector as a function of an

[6] The derivation of the optimal hours and wage equations follows exactly from Shimer (2010). The full derivation is therefore omitted for brevity

optimal target wage (which takes into consideration that the wage may be fixed for a lengthy period) and the average wage in the previous period.

The complete staggered wage problem can be summarized as

$$\begin{aligned} w_j^0 =& \frac{1-\tau_L}{1+\tau_p}(1-\eta)\left[p^n(\nu_j)A_j h_j(1-\bar{z}(\nu_j))\right] \\ & \eta\left[\frac{\psi}{\lambda}\frac{h^{1+\chi}}{1+\chi}+\bar{p}b+\frac{\bar{\phi}_j}{\lambda}\right] \\ w_j^d =& h_j+\rho_j(1-\pi_j)E\left[Q(w_j^d)'\right] \\ w_j^s(1-\tau_L)w_j^d =& w_j^0+\rho_j(1-\pi_j)E\left[Q(w_j^s)'(1-\tau_L)(w_j^d)'\right] \\ w'_j =& (1-\rho_j)(w_j^s)'+\rho_j w_j \end{aligned} \tag{23}$$

If $\rho_j = 0$, the wage problem simplifies to the fully flexible wage problem.[7]

1.5. Government

The government runs a balanced budget such that revenues exactly equal expenditures,

$$\sum_j(\tau_L+\tau_P)n_j h_j w_j+\tau^e e_j=\sum_i u_i\bar{p}b+T+\sum_j p_j g_j \tag{24}$$

where g_j represents government spending on good j.

1.6. Market Clearing

Markets clear in each sector when net output equals total demand,

$$y_j = c_j + g_j \tag{25}$$

The numerical model finds gross prices p_j such that each market clears each period.

1.7. Calibration

[7] Unlike Gertler and Trigari (2009), we allow for hours variation at each firm. We continue to assume that hours are set to maximize the joint surplus of a match. As a result, hours are equal across firms within a sector regardless of a given wage contract.

1.7..1 Households

The benchmark calibration of EDGE in Hafstead and Williams III (2018) assumes an average annualized rate of return of 4 percent. Given monthly periods, this implies the discount factor $\beta = 0.996$. The Frisch elasticity of labor supply is $\chi = 1$. As discussed in Hall and Milgrom (2008) this value represents an approximate average between the elasticity found for middle-age men (0.7) and higher elasticities found for women and young men. Without loss of generality, the time endowment for each worker is one in each period. The disutility parameter ψ is set such that average time worked in the no-policy steady state is 0.33 (i.e., one-third of the time endowment). The tax on labor τ_L is 0.25, which is approximately the average marginal income tax rate on labor income from federal and state income taxes combined in the United States.

EDGE assumes an unemployment rate of 5 percent in the no-policy equilibrium, consistent with the long-run equilibrium rate in the United States. The replacement rate is calibrated to be consistent with the wage bargaining equation. The implied value of b implies a replacement rate of approximately 41 percent (unemployment benefits as a percentage of after-tax earnings). This value is much larger than the 25 percent value used by Hall and Milgrom (2008), but it closely matches values used in other studies (for example, **?**).

The elasticity of substitution across goods is assumed to be 0.75.

1.7..2 Firms

In the benchmark calibration of EDGE in Hafstead and Williams III (2018), there are two sectors, clean and dirty ($j \in [c, d]$). The sectors are assumed to be identical, except for their emissions rates and their relative sizes. The clean sector is assumed to produce 80 percent of the consumption good and employ 80 percent of employed workers; government demand for each good is set to zero, implying symmetric productivity parameters for both sectors. The clean sector is assumed to have zero emissions and the dirty sector emissions factor is calibrated to match an emissions rate (metric tons of energy-related carbon dioxide emissions per unit of consumption) of 0.0005.

According to the Job Openings and Labor Turnover Survey (JOLTS), the average separation rate between 2015 and 2016 was 3.5 percent per month. Both sectors have the same separation rate in Hafstead and Williams III (2018). The payroll tax τ_p is 0.12, equal to the employer + employee contributions in the United States.

The benchmark EDGE model uses the abatement cost function of Nordhaus (2008), $z(\nu_j) =$

$a_1 \nu_j^{a_2}$. The curvature parameter a_2 is set to 2.8 as in Nordhaus (2008).[8] Given the unit scale of the labor force, a_1 equals one.[9]

1.7..3 Matching and Bargaining

Following Hall and Milgrom (2008), the benchmark value for the match elasticity γ_j is set to be 0.5 for both sectors. Bargaining power η is also set equal to 0.5, a standard assumption in the search model literature. The matching efficiency parameter μ_j is calibrated following the method of Shimer (2010). Recruiters are assumed to be able to recruit 25 workers per unit of time ($q_j = 25$); this assumption is consistent with findings from Silva and Toledo (2009) that the cost of recruiting a worker is approximately 4 percent of one worker's quarterly wage. The match elasticity is then simply a function of recruiter productivity and the steady state value of labor market tightness.

The benchmark calibration assumes fully flexible wages ($\rho = 0$) and no industry switching costs $\xi = 1$. Hall and Milgrom (2008) also considers staggered wage contracts with average contract lengths of 9 and 27 months (i.e. $1/(1-\rho) = 9$) and industry switching frictions with $\xi = 0.25$ and $\xi = 0.1$.

[8] This functional form is common in the literature. However, note that it violates the assumption assumption that $\lim_{x \longrightarrow 1} \bar{z}(x) = \infty$ (i.e., that the cost of abatement goes to infinity as emissions net of abatement goes to zero). That would cause problems at high levels of the emissions tax rate (for a sufficiently high tax rate, firms would set abatement high enough that net emissions would be negative). But the function generates reasonable results for the tax rates considered in Hafstead and Williams III (2018).

[9] This implies that the dirty sector can reduce emissions by 19.31 percent at the expense of 1 percent of its gross output

2. Social Accounting Matrix Framework

In the first extension of EDGE beyond the benchmark model, a social accounting matrix (SAM) framework is added to make the model comparable with large-scale economy-wide computable general equilibrium models. The SAM framework introduces intermediate inputs into production and removes the abatement function used in the benchmark model. Emissions in the model are associated with the consumption (by firms, households, or government) of energy goods (the emissions rate can vary across energy goods). Nested CES consumption and production functions are then used to determine how relative demand between energy goods, between energy goods and non-energy goods, and between intermediate inputs and the labor input (in the case of production demand) change in response to changing relative prices. International trade is also added to the model by introducing foreign sectors that demand domestically produced goods (exports) and supply foreign produced goods (imports). Domestic households and firms may demand either domestically produced goods or foreign produced goods (imports). The model uses standard Armington assumptions for relative demand of domestically produced goods or foreign produced goods.

This section will not fully document the entire EDGE-SAM model, but will rather only denote where changes have been made relative to EDGE-BENCHMARK.

2.1. Matching and Bargaining

There are no differences in the matching or bargaining processes between EDGE-BENCHMARK and EDGE-SAM.

2.2. Households

The household problem is largely unchanged. In EDGE-SAM, the consumption good c_j is a CES aggregate of domestically produced goods or foreign produced goods, c_j^d and c_j^f. The household chooses the fraction of its consumption of each good to be supplied by domestic/foreign producers to minimize the cost of the aggregate good c_j. The price of the consumption good $\hat{p}_j^c$ is a function of the domestic price p_j^d, the foreign price p_j^f, and the exchange rate e,

$$\hat{p}_j^c = (\gamma^f)^{\frac{\sigma^f}{1-\sigma^f}} \left[(\alpha_j^f)^{\sigma^f} (p_j^d)^{1-\sigma^f} + (1-\alpha_j^f)^{\sigma^f} (p_j^f/e)^{1-\sigma^f} \right]^{\frac{\sigma^f}{1-\sigma^f}}, \tag{26}$$

where σ^f represents the Armington elasticity of substitution between domestic and foreign goods. The optimal domestic and foreign consumption demand shares are

$$\begin{aligned}\frac{c_j^d}{c_j} =& (\gamma^f \alpha^f)^{\sigma^f} \left[\frac{p_j^d}{\hat{p}_j^c}\right]^{-\sigma^f} \\ \frac{c_j^f}{c_j} =& (\gamma^f (1-\alpha^f))^{\sigma^f} \left[\frac{p_j^f/e}{\hat{p}_j^c}\right]^{-\sigma^f} .\end{aligned} \tag{27}$$

Equations for the price of the aggregate consumption good and demand for each consumption good are similar to EDGE-BENCHMARK - the price $\hat{p}_j^c$ replaces the (domestic) price p_j in each equation,

$$\bar{p} = (\gamma^c)^{\frac{\sigma^c}{1-\sigma^c}} \left[\sum_j (\alpha_j^c)^{\sigma^c} \hat{p}_j^{1-\sigma^c}\right]^{\frac{\sigma^c}{1-\sigma^c}} \tag{28}$$

and

$$c_j = (\gamma^c \alpha_j^c)^{\sigma^c} \left[\frac{\hat{p}_j}{\bar{p}}\right]^{-\sigma^c} \bar{C}. \tag{29}$$

2.3. Firms

EDGE-SAM uses a nested CES production function for private sector production. Output is a function of labor and intermediate inputs,

$$y_j = f(h_j n_j (1-\bar{v}_j), \bar{I}_j) \tag{30}$$

where h_j, n_j and $\bar{v}_j$ are hours per worker, number of workers, and recruitment ratio, respectively, exactly as in EDGE-BENCHMARK.

Here $\bar{I}_j$ represents intermediate inputs. Sectors are split into energy and material (non-energy) producing sectors. Let $\bar{E}_j$ denote total demand for energy, $\bar{M}_j$ denote total demand for material goods, $E_{i_e j}$ denote demand for energy good i_e by sector j, and $M_{i_m j}$ denote demand for material good i_m.

Using energy goods as an example, at the lowest level of the nest, the input of good i_e to sector j is an aggregate of domestically and foreign produced good i_e. As the case for consumption, the price of the input good $\hat{p}_{i_e j}^e$ for sector j is a function of the domestic price $p_{i_e}^d$, the foreign price $p_{i_e}^f$, and the exchange rate e,

$$\hat{p}_{i_e j}^e = (\gamma^f)^{\frac{\sigma^f}{1-\sigma^f}} \left[(\alpha_{i_e j}^f)^{\sigma^f} (p_{i_e}^d)^{1-\sigma^f} + (1-\alpha_{i_e j}^f)^{\sigma^f} (p_{i_e}^f/e)^{1-\sigma^f}\right]^{\frac{\sigma^f}{1-\sigma^f}} , \tag{31}$$

and

$$\begin{aligned}\frac{E^d_{i_ej}}{E_{i_ej}} =&(\gamma^f\alpha^f_{i_ej})^{\sigma^f}\left[\frac{p^d_{i_e}}{\hat{p}^e_{i_ej}}\right]^{-\sigma^f}\\ \frac{E^f_{i_ej}}{E_{i_ej}} =&(\gamma^f(1-\alpha^f_{i_ej}))^{\sigma^f}\left[\frac{p^f_{i_e}/e}{\hat{p}^e_{i_ej}}\right]^{-\sigma^f}.\end{aligned} \tag{32}$$

Identical equations hold for material good demands.

The price of the energy aggregate $\hat{p}^{\bar{e}}_j$ is a function of the energy good composite prices $\hat{p}^e_{i_ej}$ and potential emissions taxes τ^e,

$$\hat{p}^{\bar{e}}_j = (\gamma^e)^{\frac{\sigma^e}{1-\sigma^e}}\left[\sum_j(\alpha^e_j)^{\sigma^e}(\hat{p}^e_{i_ej}+\tau^e\mu^e_{i_ej})^{1-\sigma^e}\right]^{\frac{\sigma^e}{1-\sigma^e}} \tag{33}$$

and relative demand is given by

$$\frac{E_{i_ej}}{\bar{E}_j} = (\gamma^e\alpha^e_j)^{\sigma^e}\left[\frac{\hat{p}^e_{i_ej}}{\hat{p}^{\bar{e}}_j}\right]^{-\sigma^e}, \tag{34}$$

where σ^e is the elasticity of demand across different energy goods. $\mu^e_{i_ej}$ is the emissions factor for the use of energy good i_e by sector j. Again, identical equations hold for material goods ($\mu^e_{i_m} = 0$ by definition).

Finally, price of the intermediate good $\bar{p}^I_j$ for sector j is a composite of the energy and material composites,

$$\bar{p}^I_j = (\gamma^I)^{\frac{\sigma^I}{1-\sigma^I}}\left[(\alpha^I_j)^{\sigma^I}(\hat{p}^{\bar{e}}_j)^{1-\sigma^I}+(1-\alpha^I_j)^{\sigma^I}(\hat{p}^{\bar{m}}_j)^{1-\sigma^I}\right]^{\frac{\sigma^I}{1-\sigma^I}}, \tag{35}$$

and relative demand is given by

$$\begin{aligned}\frac{\bar{E}_j}{\bar{I}_j} =&(\gamma^I\alpha^I_j)^{\sigma^I}\left[\frac{\hat{p}^{\bar{e}}_j}{\bar{p}^I_j}\right]^{-\sigma^I}\\ \frac{\bar{M}_j}{\bar{I}_j} =&(\gamma^I(1-\alpha^I_j))^{\sigma^I}\left[\frac{\hat{p}^{\bar{m}}_j}{\bar{p}^I_j}\right]^{-\sigma^I}.\end{aligned} \tag{36}$$

The dynamic problem of the firm is represented by the following Bellman equation,

$$J(n_j) = \max_{\bar{v}_j,\bar{I}_j}\left[p^d_jf(h_jn_j(1-\bar{v}_j),\bar{I}_j)-(1+\tau_p)n_jh_jw_j-\bar{p}^I_j\bar{I}_j+E\left[QJ(n_j^{'})\right]\right] \tag{37}$$

subject to

$$n_j^{'} = (1 - \pi_j)n_j + q_j \bar{v}_j h_j n_j \tag{38}$$

Firms set total aggregate intermediate inputs to satisfy the first-order constraint,

$$p_j^d f_I = \bar{p}_j^I \tag{39}$$

The first-order condition for the recruitment ratio is

$$p_j^d F_{L,j} = q_j E\left[QJ_{n_j}^{'}\right] \tag{40}$$

where $p_j^d F_{L,j}$ represents the marginal value of an additional hour of production (workers can be costlessly moved between production work and recruiting, so the marginal cost of an hour spent on recruiting is one fewer hour of production work), and denotes the current period value of an additional worker in the following period. The marginal value of a worker is

$$J_{n_j} = p_j^d F_{L,j} h_j - (1 + \tau_p) h_j w_j + (1 - \pi_j) E\left[QJ_{n_j}^{'}\right] \tag{41}$$

2.4. Government

The government must combine labor and intermediate goods to produce a fixed level of public goods, g each period. The government production function is

$$g = f_g(h_g n_g (1 - \bar{v}_g), \bar{I}_g) \tag{42}$$

where $h_g n_g (1 - \bar{v}_g)$ denotes the level of labor input to government production and $\bar{I}_g$ denotes the aggregate intermediate input to the government sector. The choice of intermediate inputs determines the unit cost of the intermediate input, $\bar{p}_g^{\bar{I}}$, and the government faces the same nested intermediate input function as the private sector, as described above.

In the EDGE-SAM, the government inherits a stock of workers each period and allocates workers to production or recruitment. Similar to the private firm problem, the allocation of workers between production and recruitment introduces a dynamic decision into the government problem. The problem of the government can be written as

$$G(n_g) = \max_{\bar{v}_g, \bar{I}_g} \left[-(1 + \tau_p) h_g n_g w_g - \bar{p}_g^I \bar{I}_g + E\left[QG(n_g^{'})\right]\right] \tag{43}$$

subject to

$$n_g^{'} = (1 - \pi_g) n_g + q_g \bar{v}_g h_g n_g \tag{44}$$

and the production constraint

$$f_g(h_g n_g(1-\bar{v}_g), \bar{I}_g) \geq g \tag{45}$$

The government takes as given the endogenous recruiter productivity q_g and hours and wages follow an identical bargaining process to the private sector. The first-order condition with respect to government recruitment is

$$\lambda_g F_{L,g} = q_g E\left[QG'_{n_g}\right] \tag{46}$$

Here, λ_g represents the Lagrange multiplier on the production constraint. The marginal value of an additional worker is

$$G_{n_g} = \lambda_g F_{L,g} h_g - (1+\tau_p) h_g w_g + (1-\pi_g) E\left[QG'_{n_g}\right]. \tag{47}$$

Additional workers are valuable to the government to the extent that they ease the constraint that public goods production must be greater or equal to g.

2.5. Foreign Economy

A single foreign economy represents the rest of the world (ROW). The foreign economy mirrors the domestic economy in all respects, although the scale of the ROW economy is larger.

The price of foreign goods is denoted by p_i^f. The exchange rate e converts domestic currency into the foreign currency such that the prices for domestic goods faced by the foreign agents are $p_i^f e$ and the prices for foreign goods faced by domestic agents are p_i^f/e. EDGE-SAM assumes balanced trade, with the exchange rate adjusting such that the values of imports and exports are equal in each period, as defined in the discussion of market-clearing below. Exports are the sum of foreign demand for domestically produced goods and imports are the sum of domestic demand for foreign-produced goods.

For energy goods, total domestic demand for foreign-produced goods is

$$im_{i_e} = c_{i_e}^f + \sum_j E_{i_e j}^f + E_{i_e g}^f \tag{48}$$

and foreign demand for domestically produced goods is

$$ex_{i_e} = c_{i_e}^{fd} + \sum_j E_{i_e j}^{fd} + E_{i_e g}^{fd} \tag{49}$$

where the superscript fd refers to foreign demand for domestic products. Similar equations hold for material goods.

2.6. Market Clearing

Markets clear when supply equals demand. For energy goods, markets clear when

$$y_{i_e} = c^d_{i_e} + \sum_j E^d_{i_e j} + E^d_{i_e g} + ex_{i_e} \tag{50}$$

Similar equations hold for material goods. The numerical model finds gross prices p^d_i and p^f_i such that each market clears each period.

The model also imposes a balanced trade requirement each period such that the total value of exports must equal the total value of imports,

$$\sum_i p^d_i ex_i = \sum_i (p^f_i / e) im_i \tag{51}$$

The exchange rate e adjusts each period to ensure balanced trade. Finally, there is no market for public goods, but the constraint on the provision of the public good introduces a market-clearing-like condition: the shadow price λ_g is "market-clearing" if and only if the constraint $g = f_g(h_g n_g(1 - \bar{v}_g), \bar{I}_g)$ is binding. An identical condition holds for the foreign government with shadow price λ^f_g.

2.7. Data and Calibration

2.7..1 Data

In Hafstead et al. (2022), EDGE-SAM is calibrated to 2015 data. Data on input use by sector, consumption by households, and labor input by sector are aggregated from the 2015 Bureau of Economic Analysis (BEA) Make and Use Tables from the Annual Industry Accounts. Industry-specific separation rates are derived by averaging monthly total separation rates for each industry grouping in the Job Opening and Labor Turnover Survey from the Bureau of Labor Statistics. In Hafstead et al. (2022), EDGE-SAM had 22 sectors. Table <xx> displays the list of industries and separation rates.

Emissions coefficients are calibrated to match energy-related carbon dioxide emissions data from the Energy Information Administration (EIA). Updated data or changes in

Also in Hafstead et al. (2022), the foreign economy is symmetric to the domestic economy, but is three times larger than its domestic counterpart.

2.7..2 Calibration

The calibration of common parameters is identical between EDGE-BENCHMARK and EDGE-SAM. The hours of public sector hours are fixed $h_g = 1/3$. Conditional on the labor market parameters and this assumption, a calibration procedure is used to solve for a common disutility of work parameter ψ, the level of unemployment benefits b, the match efficiency parameter by sector μ_j, and hours per worker in private sector h_j that are consistent with the model equations and data (including asymmetric separation rates across sectors).

3. Job Quality Extension

To capture differences in job quality, EDGE adds two features: on-the-job search (OJS) and a job ladder or promotion process (JL). EDGE-OJS introduces on-the-job search with differences in job "quality" across industries. In EDGE-BENCHMARK, largely symmetric industries implies that workers are basically indifferent between working in either industry, both before and after the implementation of environmental policies. By calibrating differences in recruiter productivity (measured directly through q_j or indirectly through μ_j), EDGE introduces important differences in V_{n_j} across sectors: workers value having jobs in industries where jobs are difficult to fill because they earn a higher surplus of match. On-the-job search allows workers in jobs with relatively low value to search for jobs in high value sectors without having to experience an unemployment spell, where as workers in high value sectors will never participate in on-the-job search.

The EDGE-JL model introduces a promotion process within each industry. Ex-ante identical workers start in an industry as associate workers and may be promoted to tenured workers with some exogenous probability each period. Tenured workers have both a higher productivity and a lower exogenous quit rate than associate workers.

Finally, EDGE-JQ combines both features into a single model. Rather than individually describe each model, this section will describe the combined model with both features. Both the OJS and JL models can be added to either the EDGE-BENCHMARK model with abatement costs or the EDGE-SAM model with intermediate inputs into production. Here, the abatement cost production model is used, without loss of generality.

Again, this section will not fully document the entire EDGE-JQ model, but will rather only denote where changes have been made relative to EDGE-BENCHMARK.

3.1. Matching Process

As in the EDGE-BENCHMARK model, all workers are ex-ante homogeneous. Workers can be either employed or unemployed, and employed workers in a given sector j can be either associate workers n_j^a or tenured workers n_j^t. Total employment is $\bar{n} = \sum_j (n_j^a + n_j^t)$ and the measure of unemployed workers, which is equivalent to the unemployment rate, is $\bar{u} = 1 - \bar{n} = \sum_i u_i$, where again u_i denotes the number of unemployed workers previously employed in sector i.

All new entrants start as associate workers. Each period, some associate workers can become tenured workers with probability ι. Tenured workers are more productive and have lower sep-

aration rates. All workers, when separated, go back to general unemployment./[10]

All unemployed workers and some associate employed workers search for jobs. As standard in the literature, we assume that unemployed search is costless, but on-the-job search is costly to searchers (in terms of utility). Let s_j denote the fraction of associate workers engaged in on-the-job search. The cost of search is a strictly increasing and convex function of the fraction of workers engaged in searching, $k(s_t)$ with $k(0) = 0$, $k' > 0$ and $k'' > 0$. Convexity of the cost of on-the-job search guarantees uniqueness of the fraction of workers seeking outside employment, with $s_j n_j^a$ denoting the number of employed searchers.

Let v_j denote recruitment effort in each sector j; one unit of recruitment effort can hire $q_j^a(\bar{\Theta})$ workers.[11]

The EDGE-OJS+JL generalized matching function, is similar to the EDGE-BENCHMARK model's one but takes into account that there are more job searchers, and is given by

$$m_{ij} = \mu_j v_j \bar{s}_i \left[\xi \left(\sum_k v_k \right)^{-\gamma_j} \left(\sum_k \bar{s}_k \right)^{\gamma_j - 1} + (1-\xi)(v_j)^{-\gamma_j} (\bar{s}_i)^{\gamma_j - 1} \delta_{ij} \right] \tag{52}$$

where μ_j and γ_j are the matching efficiency and matching elasticity parameters for each sector, ξ is a parameter that controls for the degree of matching across sectors (see 1.), and $\bar{s}_i = u_i + s_i n_i^a$ is the number of searchers (unemployed and employed) with experience in sector i.

As in EDGE-BENCHMARK, there are three measures of labor market tightness - $\theta_{ij} = \frac{v_j}{\bar{s}_i}$ is the ratio of recruitment effort in sector j to unemployed workers from sector i and employed searchers from sector i, $\theta_j = \frac{v_j}{\sum \bar{s}_i}$ is the ratio of recruitment effort in sector j to the total number of searchers, and $\bar{\theta} = \sum_j \theta_j$ is the ratio of total recruitment effort to the total number of job searchers.

Recruiting productivity q_j^a is defined as the number of matches per recruitment effort, $q_j^a = \sum_i m_i j/(v_j)$; the job finding probability ϕ_{ij} is defined as the number of matches per searchers, $\phi_{ij} = m_{ij}/(\bar{s}_i)$. Recruiting productivity and job finding probabilities are functions of labor market tightness,

[10] The model does not distinguish between formally tenured and associate workers, implying that unemployed tenured workers have the same job prospects as unemployed associate workers.

[11] Unlike EDGE-BENCHMARK, v_j is recruitment effort and not the number of recruiters. As described in the Firms section, recruitment and production are done by a combination of associate and tenured workers.

$$q_j^a = \mu_j \left[\xi \bar{\theta}^{\gamma_j} + (1-\xi)\theta_{ii}^{-\gamma_j} \right] \tag{53}$$

$$\phi_{ij} = \mu_j \left[\xi \theta_j \bar{\theta}^{-\gamma_j} + (1-\xi)\delta_{ij}\theta_{ij}^{1-\gamma_j} \right] \tag{54}$$

3.2. Households

The household problem differs from the EDGE-BENCHMARK household problem because there are two type of workers in the utility function, there are two type of worker's law of movement, and associate workers are allowed to search for jobs.

The lifetime discounted utility of the household depends on the distribution of worker-types across sectors and unemployed workers across sectors and can be written as a Bellman equation:

$$\begin{aligned} V(B, n_j^a, n_j^t, u_i) = \max_{\bar{C}, B, s_j} [& \sum_j n_j^a (U(\bar{C}, h_j^a) - k(s_j)) + \sum_j n_j^t (U(\bar{C}, h_j^t)) \\ & + \sum_i u_i U(\bar{C}, 0) + \beta E \left[[V(B', n_j^{a'}, n_j^{t'}, u_i'] \right] \end{aligned} \tag{55}$$

subject to the budget constraint,

$$\bar{p}\bar{C} + QB' \leq \sum_j ((1-\tau_L) n_j^a w_j^a h_j^a + (1-\tau_L) n_j^t w_j^t h_j^t) + \sum_i u_i \bar{p} b + B + T \tag{56}$$

and the laws of motion for unemployment, associate employment and tenure employment by sector:

$$\begin{aligned} u_i' &= \pi_i^a n_i^a + \pi_i^t n_i^t + u_i (1 - \sum_j \phi_{ij}) \\ n_j^{a'} &= (1 - \pi_j^a - \sum_k \phi_{jk} s_j - \iota) n_j^a + \sum_k \phi_{kj} (u_k + s_k n_k^a) \\ n_j^{t'} &= (1 - \pi_j^t) n_j^t + \iota n_j^a \end{aligned} \tag{57}$$

where β is discount factor, τ_L is the tax rate on labor income, T is government lump-sum transfers (taxes if negative), and π_j^a and π_j^t are the exogenous rate of job destruction in sector j for each type in each period, and ι is the probability that an associate worker become promoted to tenure worker.

The first-order condition with respect to search effort s_j is given by

$$k_j' = \beta \sum_k \phi_{jk} V_{n_k}' - \beta \sum_k \phi_{jk} V_{n_j}' \tag{58}$$

The marginal value of having an associate worker employed in a given sector is given by the envelope condition with respect to the number of associate workers employed in that sector,

$$\begin{aligned} V_{n_j^a} =& U(\bar{C}, h_j^a) - k(s_j) + \lambda(1-\tau_L) w_j^a h_j^a \\ &+ (1 - \pi_j^a - \sum_k \phi_{jk} s_j - \iota)\beta V_{n_j^a}' + \iota \beta V_{n_j^t}' + \sum_k \phi_{jk} s_j \beta V_{n_k^a}' + \pi_j^a \beta V_{u_j}' \end{aligned} \tag{59}$$

The marginal value of having a tenure worker employed in a given sector is given by the envelope condition with respect to the number of tenured workers employed in that sector,

$$V_{n_j^t} = U(\bar{C}, h_j^t) + \lambda(1-\tau_L) w_j^t h_j^t + (1-\pi_j^t)\beta V_{n_j^t}' + \pi_j^t \beta V_{u_j}' \tag{60}$$

The marginal value of having an unemployed worker with previous experience in a given sector is given by the envelope condition with respect to the the number of unemployed workers with experience in that sector,

$$V_{u_j} = U(\bar{C}, 0) + \lambda \bar{p} b + \beta V_{u_j}' + \beta \sum_k \phi_{jk} (V_{n_k^a}' - V_{u_j}') \tag{61}$$

3.3. Firms

As in EDGE-BENCHMARK, the basic problem of the firm is to choose recruitment and abatement, conditional on its stock of workers, to maximize the value of the firm over time. Production and recruitment are done by both types of workers. The fraction of workers dedicated to recruitment, $\bar{v}_j$ is assumed to be equal across worker types - $v_j = f(\bar{v}_j A_j^a h_j^a n_j^a, \bar{v}_j A_j^t h_j^t n_j^t)$ where f is a CES function that combines effort from both types of workers. Gross output is a function of production labor input, l_j, $y_j = A_j l_j$, where production labor is $l_j = f(A_j^a h_j^a n_j^a (1-\bar{v}_j), A_j^t h_j^t n_j^t (1-\bar{v}_j)$.

As in EDGE-BENCHMARK, net output is equal to gross output less abatement costs and can be written as $\bar{y}_j = A_j f(A_j^a h_j^a n_j^a (1-\bar{v}_j), A_j^t h_j^t n_j^t (1-\bar{v}_j)(1-\bar{z}(\nu_j))$. Firms take the endogenous recruiting productivity q_j^a as given when making recruiter decisions. The dynamic problem of the firm is to choose the recruiter ratio and abatement to maximize the value of the firm.

$$\begin{aligned} J(n_i^a, n_i^t) = \max_{\bar{v}_j, \nu_j} \{ & p_j^n(\nu_j) A_j f(A_j^a h_j^a n_j^a (1-\bar{v}_j), A_j^t h_j^t n_j^t (1-\bar{v}_j)(1-\bar{z}(\nu_j)) \\ & - (1+\tau_p) n_j^a h_j^a w_j^a - (1+\tau_p) n_j^t h_j^t w_j^t + E\left[Q J(n_j^{a'}, n_i^{t'}) \right] \} \end{aligned} \tag{62}$$

subject to

$$\begin{aligned} n_i^{a'} &= (1-\pi_i^a-\rho_j\phi_j s_i-\iota)n_i^a + q_j^a f(\bar{v}_j A_j^a h_j^a n_j^a, \bar{v}_j A_j^t h_j^t n_j^t) \\ n_i^{t'} &= (1-\pi_i^t)n_i^t + \iota n_i^a \end{aligned} \tag{63}$$

where p_j^n denotes the net price received by sector j for its output and τ_p denotes the payroll tax. The net price of output is equal to the price of the good minus the emissions tax payments, $p_j^n(\nu_j) = p_j - \tau^e \mu_j^e (1-\nu_j)$.

Firms add recruiters (from both pools of worker types) until the marginal cost of additional recruiting equals the benefit of recruiting new associate workers for the following period; the first-order condition is

$$p_j^d A_j(1-\bar{z}(\nu_j))[f_{a_j} A_j^a h_j^a n_j^a + f_{t_j} A_j^t h_j^t n_j^t] = q_j^a [f_{a_j} A_j^a h_j^a n_j^a + f_{t_j} A_j^t h_j^t n_j^t] E\left[QJ'_{n_j^a}\right] \tag{64}$$

where f_{a_j} and f_{t_j} are the derivatives of the aggregation function with respect to each labor input; the expression simplifies to the same expression as in the EDGE-BENCHMARK model

$$p_j^d A_j(1-\bar{z}(\nu_j)) = q_j^a E\left[QJ'_{n_j^a}\right] \tag{65}$$

From the envelope condition with respect to the number of associate workers, the marginal value of an additional associate worker for a firm j:

$$\begin{aligned} J_{n_j^a} =& p_j^n A_j A_j^a h_j^a (1-\bar{z}(\nu_j)) f_{a_j} - (1+\tau_p) w_j^a h_j^a + (1-\pi_j^a - \sum_k \phi_{jk} s_j - \iota) E\left[QJ'_{n_j^a}\right] \\ &+ \iota E\left[QJ'_{n_j^t}\right] \end{aligned} \tag{66}$$

Similarly, the envelope condition with respect to the number of tenure workers, the marginal value of an additional tenure worker for firm j:

$$J_{n_j^t} = p_j^n A_j A_j^t h_j^t (1-\bar{z}(\nu_j)) f_{t_j} - (1+\tau_p) w_j^t h_j^t + (1-\pi_j^t) E\left[QJ'_{n_i^t}\right] \tag{67}$$

3.4. Wage Bargaining

The model again assumes that wages and hours are set through a Nash bargaining process for both types of workers.[12] Let η^i denote the bargaining power of the employer for worker type

[12] The model also assumes that the outside option for tenured workers is to become unemployed rather than become an associate.

$i \in a, t$. The equilibrium wages and hours are the solution to

$$\max_{w_j^i, h_j^i} J_{n_j^i}^{\eta^i} [V_{n_j^i} - V_{u_j}]^{1-\eta^i}. \tag{68}$$

As in EDGE-BENCHMARK, hours per worker for each type of worker are set to maximize the value of the match surplus. This means that the marginal value of an additional hour of work is equal to the disutility of an additional hour of work,

$$(1+\tau_p)\psi^i (h_j^i)^\chi = (1-\tau_L)\lambda p^n(\nu_j) A_j A_j^i (1-\bar{z}(\nu_j)) f_{i_j} \tag{69}$$

Nash bargaining also implies that the match surplus is divided according to a constant share rule. The equilibrium after-tax pay packet for an associate worker is

$$\begin{aligned}(1-\tau_L)h_j^a w_j^a =& \frac{1-\tau_L}{1+\tau_p}(1-\eta)\left[p^n(\nu_j)A_j A_j^a h_j^a (1-\bar{z}(\nu_j)) f_{a_j}\right] \\ & \eta\left[\frac{\psi^a}{\lambda}\frac{(h^a)^{1+\chi}}{1+\chi} + \frac{k(s_j)}{\lambda} + \bar{p}b + \frac{\bar{\phi}_j^a}{\lambda}\right]\end{aligned} \tag{70}$$

where $\bar{\phi}_j^a = \beta \sum_k \phi_{jk}(1-s_j)(V'_{n_k^a} - V'_{u_j})$.

The after-tax pay packet for a tenured worker is

$$\begin{aligned}(1-\tau_L)h_j^t w_j^t =& \frac{1-\tau_L}{1+\tau_p}(1-\eta)\left[p^n(\nu_j)A_j A_j^t h_j^t (1-\bar{z}(\nu_j)) f_{t_j}\right] \\ & \eta\left[\frac{\psi^t}{\lambda}\frac{(h^t)^{1+\chi}}{1+\chi} + \bar{p}b + \frac{\bar{\phi}_j^t}{\lambda}\right]\end{aligned} \tag{71}$$

where $\bar{\phi}_j^t = \beta \sum_k \phi_{jk}(V'_{n_k^a} - V'_{u_j})$. Note: In $\bar{\phi}_j^t$, it is not a typo that the expression includes $V'_{n_k^a}$, as this comes from the value of being unemployed and unemployed workers can only be hired as associates.

4. Capital Extension

Forthcoming

5. How to use the code

Th Employment Dynamics in General Equilibrium (EDGE) suite of models are solved in GAMS using the PATH solver. GAMS can be downloaded for free but the size of EDGE requires licenses for both GAMS and PATH. PATH should be set as the default solver for mcp problems.

The EDGE programs are meant to be fully self-contained. A ”MAIN” file calls the appropriate subprograms and will complete policy simulations specified within the ”MAIN” file. The sections below explain how to execute the program and briefly specifies each sub-program.

5.1. EDGE-BENCHMARK

The program EDGE-B_MAIN_LUMP.gms simulates the effect of carbon pricing policies with lump-sum rebating using the EDGE-B framework. The set ”it” determines the number of policy simulations. The program begins by defining the ”deep” model parameters as described in section 1.7.

The program then calls the subprogram EDGE-B_CAL.gms. This calibration program defines and solves for the complete benchmark equilibrium steady state, conditional on the model parameters and assumptions about the benchmark recruiter productivity, the benchmark unemployment rate, and the benchmark number of hours worked (for a single sector).

The price path for each simulation is defined in the section beginning on line 91.

The full model is defined in subprogram EDGE-B_EQN.gms. The subprogram defines the steady state and transition equations and variables. The subprogram also defines the steady state model and the ”full” model (transition and steady state) for various revenue recycling options, and initializes all variables to their benchmark steady state levels.

To verify consistency between the steady state model and calibration model, the model should be able to solve in zero iterations (since the variables should be consistent). The ”MAIN” program then solves for the steady state for the first defined policy, which is used to initialize the transition variables for the first simulation using the full model. The ”MAIN” program then iteratively solves for each policy price path, which implicitly uses the solution from the previous iteration as an initial guess for the current iteration. Output from both the steady state transition can be saved between policy simulations.

Table 1. Codes to solve EDGE Benchmark model

File Name	Description
EDGE-B_MAIN_LUMP.gms	Simulates the effect of carbon pricing policies with lump-sum rebating using the EDGE-B framework.
EDGE-B_CAL.gms	Calibrates, defines and solves for the complete benchmark equilibrium steady state, conditional on the model parameters and assumptions using the EDGE-B framework.
EDGE-B_EQN.gms	Full Model equations and variables are defined, both for the steady state and transition.

Notes: The Employment Dynamics in General Equilibrium (EDGE) suite of models are solved in GAMS using the PATH solver.

5.2. EDGE-SAM

Forthcoming

5.3. EDGE-JQ

Forthcoming

5.4. EDGE-FULL

Forthcoming

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