Dynamic Regional General Equilibrium Model - DR-GEM Documentation

Marc Hafstead

June 2022
About the Author

Marc Hafstead is a Fellow at Resources for the Future (RFF). He joined RFF in 2013 from Stanford University. He is also the director of the Carbon Pricing Initiative and Climate Finance and Financial Risk Initiative at RFF.

Acknowledgements

Financial support for this model project has been provided by a number of funders over the years, including RFF’s Carbon Pricing Initiative, the State of Vermont, the Environmental Defense Fund, and the Sloan Foundation.
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Overview

The Dynamic Regional General Equilibrium Model (DR-GEM) is a dynamic multi-region and multi-industry intertemporal computable general equilibrium (CGE) model of the US economy with international trade. CGE models combine detailed economic data with formulas that describe economic behavior to project how an economy will respond to a policy over time. For each policy scenario, the model calculates the changes in the supply and demand of producer and consumer goods by households and firms in each region and the corresponding changes in market-clearing prices.

DR-GEM shares many features with the Goulder-Hafstead Energy-Environment-Economy (E3) model.\(^1\) Each regional economy is modeled as a collection of forward-looking agents: firms representing distinct industries within that region, a single representative household for that region, and regional and federal governments. The model captures the interactions among agents both within and across regions and solves for market-clearing prices in each period. Each agent has perfect foresight, and the model is solved in each period until it converges to a new steady-state balanced-growth equilibrium.

DR-GEM version 1.0 utilizes a national market assumption for trade across domestic regions. Producers in each region can choose between supplying their own market and a national market. Consumers (both producers and households) can choose to demand goods from either their own market or the national market. Prices are solved to clear both the regional markets and the national market.

DR-GEM originally used regional social accounting matrix (SAM) from the IMPLAN group, IMPLAN (2017), and the most up-to-date code shows how this data is utilized. However, the IMPLAN data is proprietary and cannot be used in open source codes. Work is currently being done to utilize regional social accounting matrices from the Wisconsin National Data Consortium (WiNDC).

Regional social accounting matrices (SAMs) provide information on market flows and non-market financial flows among firms, consumers, and the government. The data is augmented with information on production, physical consumption, and total expenditures by energy good from the US Energy Information Administration’s State Energy Data System, EIA (2018a), EIA (2018b), EIA (2018c). The current regional data set (not included) contains 60 industries and 51 regions (all states plus the District of Columbia). However, computational constraints limit the practical number of industries and regions that can be used at any one time. A practical number of industries and regions would be 18 and 4, respectively.

\(^1\)For a complete description of the E3 model, see Goulder and Hafstead (2017).
1. Model Structure

DR-GEM is a close cousin of the Goulder-Hafstead E3 model. The following model description closely follows Goulder and Hafstead (2017).

1.1. Goods and Trade

Let $g_i$ denote the demand for a good produced by sector $i$. A good consumed in domestic region $r_k$ could be produced by the producer of good $i$ in region $r_k$, it could be imported by a similar producer in a different region $r_l$ where $l \neq k$, or it could be imported from an international producer.

Demand for goods produced in different regions follows a simple nested CES Armington structure. Demand for domestic good $g_i$, $g^d_i$, is a mixture of good $i$ produced in the home region, $g^r_i$ and goods supplied to the "national market" $g^n_i$ (see below for definition of supply to national markets); demand for own-region and national goods is a function of relative prices of each good, $p$ and $p^n$ and the elasticity of substitution between own-region and national goods. Demand for $g_i$ is a mixture of demand for the domestic composite good $g^d_i$ and the foreign supplied good $g^f_i$. The prices of foreign supplied goods $p^f_i$ are exogenous in DR-GEM, but there is an endogenous exchange rate $exch$ that balances international trade each period such that the real value of the trade balance is constant each period; the price of good $g^f_i$ to a domestic consumer is $p^f_i / exch$. Let $p^g$ denote the optimal unit price for the optimal basket of goods from different regions (domestic and international).

Supply of goods to different regions will be explained below.

1.2. Households

DR-GEM uses a representative household framework. Each region $r$ contains a single representative household designed to capture key aspects of consumer behavior, including the choice between work and leisure, the choice between savings and consumption, and the choice of consumption expenditure across various consumer goods and services, with a focus on expenditures on energy goods.

Households choose consumption and leisure, and savings each period to maximize their in-

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2 See Appendix A for relative demand of goods using a standard CES aggregation function.
tertemporal utility subject to a budget constraint. In year $t$ the household chooses a path of "full consumption" $C(r)$ to maximize

$$U_t(r) = \sum_{s=t}^{\infty} (1 + \beta)^{t-s} \frac{\sigma}{\sigma - 1} C_s(r) \frac{\sigma - 1}{\sigma}$$  \hspace{1cm} (1)$$

where $\beta$ is the subjective rate of time preference and $\sigma$ is the intertemporal elasticity of substitution in full consumption. $C(r)$ is a CES composite of consumption of goods and services, $\bar{C}(r)$, and leisure, $l(r)$:

$$C_s(r) = \left[ \bar{C}_s(r) \frac{\sigma - 1}{\sigma} + \alpha_l l_s(r) \frac{\sigma - 1}{\sigma} \right] \frac{1}{\sigma - 1}$$  \hspace{1cm} (2)$$

where $\nu$ is the elasticity of substitution between goods and leisure and $\alpha_l$ is a leisure intensity parameter.

The household’s intertemporal budget constraint is

$$W_{t+1}(r) - W_t(r) = \tilde{r}_t(r)W_t(r) + Y^l_t(r) + GT^f_t(r) + GT^r_t(r) - GL^f_t(r) - GL^r_t(r) + BOP_t(r) + ADJ_t(r) - \bar{p}_t(r)\bar{C}_t(r)$$  \hspace{1cm} (3)$$

where $W_t$ is the household’s financial wealth at time $t$, $\tilde{r}$ is the nominal after-tax return on the household’s financial wealth holdings, $Y^l_t$ is the household’s after-tax labor income, $GT$ is transfer income from the federal or regional government, $GL$ is lump-sum taxes from the federal or regional government, $BOP_t(r)$ is the region’s international balance of payments, $ADJ_t(r)$ is the region’s domestic balance of payments, and $\bar{p}$ is the unit price of the consumption composite $\bar{C}$.

Household labor income, $Y^l_t$, is equal to total time worked (total potential labor time, $\tilde{l}$, minus leisure), times the net-of-tax wage rate, $\bar{w} = (1 - \tau_l)w$. The household maximizes utility by choosing $\bar{C}_s$, $l_s$, and $W_{s+1}$. The Lagrangian for the household problem is (ignoring $r$ for clarity)
\[
L^C = \sum_{s=t}^{\infty} (1 + \beta)^{t-s} \frac{\sigma}{\sigma - 1} \left[ \frac{v-1}{C_s^v} + \alpha^1 l_s^{\frac{v-1}{v}} \right]^{\frac{\sigma - 1}{\sigma}} + \\
\sum_{s=t}^{\infty} \lambda_{C,s} \left[ (1 + \bar{r}_s)W_s + \bar{w}(l_s - l_s) + G^T_s - W_{s+1} - \bar{p}_s \bar{C}_s \right].
\]

The first-order conditions with respect to \( \bar{C}_s, l_s, \) and \( W_{s+1} \) are

\[
\frac{\partial L^C}{\partial C_s} : (1 + \omega)^{t-s} \left[ \frac{v-1}{C_s^v} + \alpha^1 l_s^{\frac{v-1}{v}} \right]^{\frac{\sigma - 1}{\sigma}} \bar{C}_s = \lambda_{C,s} \bar{p}_s
\]

\[
\frac{\partial L^C}{\partial l_s} : (1 + \omega)^{t-s} \left[ \frac{v-1}{C_s^v} + \alpha^1 l_s^{\frac{v-1}{v}} \right]^{\frac{\sigma - 1}{\sigma}} \alpha^1 l_s^{\frac{v-1}{v}} = \lambda_{C,s} \bar{w}_s
\]

\[
\frac{\partial L^C}{\partial W_{s+1}} : \lambda_{C,s} = (1 + \bar{r}_{s+1}) \lambda_{C,s+1}
\]

The aggregate consumption good \( \bar{C}(r) \) is a composite of \( n_c \) consumption goods, \( c_1, .. c_j \). DRGEM uses a nested CES utility structure, as specified in Figure 1. At each level of the nest, households choose consumption intensities to achieve the least cost combination of goods.\(^3\)

Elasticities of substitution are common values taken from the literature. Elasticities of substitution between motor vehicle fuels and electricity and between vehicle expenditures and fuels are chosen such that the model approximates the demand response to fuel use from RFF’s transportation model (provided by Josh Linn).

By solving for optimal intensities at each level of the nest and solving for overall consumption expenditure \( \bar{C}(r) \), the model solves for the demand for each consumer good \( c_g(r) \), \( .. c_g(j) \).

Each consumer good is itself a composite of producer goods. DRGEM uses a fixed \( "G" \) matrix to map spending on consumer goods into spending on producer goods such that \( cp_j = \sum_i G_{ij} pc_i \), where \( cp_j \) is the price of consumer good \( j \), \( G_{ij} \) is the amount of spending on consumer good \( j \) that flows to producer \( i \) and \( pc_i \) is the price of consumption spending on producer good \( i \).

The model includes fixed margin requirements for personal consumer expenditures such that

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\(^3\)See Appendix A for how to solve for optimal input intensities using CES functions
$p_{ci}(r)$ is a function of the price of good $i$, $p_i^g(r)$, and the fixed margin requirements for producer good $i$ from sector $m$ (transportation or trade), $mr_{im}^c$.

$$p_{ci}(r) = \frac{p_i^g(r)}{1 + \sum_m mr_{im}^c} + \frac{\sum_m mr_{im}^c \cdot p_m^c(r)}{(1 + \sum_m mr_{im}^c)} \quad (8)$$

where $p_m^c$ is the price of providing margins from sector $m$ (see producer problem below).

Following from above, let $pce_i$ denote the personal consumption expenditure demand for good $i$, less margin requirements (that is, the flow of consumption that comes directly from good $i$):

$$pce_i(1 + \sum_m mr_{im}^c) = \sum_j G_{ij} \cdot eg_j.$$
1.3. Firms

A representative firm in each region and each sector produces a distinct output $y_i(r)$ using capital $K$, labor $l$, and intermediate inputs $IO$ (energy and materials). Producers choose variable inputs to minimize costs and choose investment to maximize its payment to the shareholders - the households.

1.3.1 Production

Output from each sector stems from a nested structure of constant-elasticity-of substitution (CES) production functions. Figure 2 displays this structure. For each of these CES nests, elasticities of substitution for each industry are taken from common estimates from the economic literature.

Variable energy inputs are a combination of (retail) electricity and non-electricity energy inputs - crude oil, natural gas (raw or distributed), coal, and petroleum refining. Variable intermediate inputs $V$ are an aggregate of the energy and material composite (which is a simple aggregate of non-energy goods).

Firms face margin requirements when purchasing certain intermediate inputs such that the price paid by industry $j$ for good $i$ is

$$p_{ij}^{io}(r) = \frac{p_i^e(r)}{(1 + \sum m r_{ijm}^{io} p_m^m(r))} + \frac{\sum m r_{ijm}^{io} p_m^m(r)}{(1 + \sum m r_{ijm}^{io})}$$

such that total expenditure for input $IO_{ij}(r)$ is $p_{ij}^{io}(r)IO_{ij}(1 + \sum m r_{ijm}^{io})$

Let $p^e$, $p^m$, and $p^v$ denote the unit prices (inclusive of margins) of the optimal composites of energy, material, and total intermediate inputs, respectively. The intermediate input composite $V$ is combined with labor $l$, which has costs $w(1 + \tau_p)$ ($\tau_p$ represents federal payroll taxes), to create a variable input composite $\bar{V}$ with unit cost $p^\bar{v}$.

For each of these CES nests, elasticities of substitution for each industry are taken from common estimates from the economic literature.

For each industry, specific intermediate inputs are considered fixed proportion inputs: the firm cannot utilize more or less of these inputs relative to other inputs (for example, crude oil input into petroleum refining). A fixed Leontief function combines these inputs with the other inputs.
$\bar{V}$ into a total variable input composite $Z$ with unit price $p^z$.

Fixed natural resources $NR$ are required to produce outputs for certain industries (for extraction industries (oil, natural gas, coal) and Electricity Generation: Nuclear, Hydro, Other). Output is a function of capital $K$, variable inputs $Z$, and natural resources $NR$: $Y = F(K, Z, NR)$. Natural resources are fixed and exogenous; the elasticity of substitution between the three inputs determines the supply elasticity for these industries. Payments for natural resources from each region are made to the households in that region at price $p_{ni}^r(r)$. For industries without natural resources inputs, output is $Y = F(K, Z)$.

$^4$If $\alpha^z$ is the share for the fixed proportion input, then $p_{ij}^z = \alpha^z p_{ij}^v + (1 - \alpha^z) p^v$. 

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1.3.2 Investment

Capital adjustment costs are modeled as the sacrifice of output associated with the process of investing in capital. Specifically, net output is equal to gross output minus adjustment costs, $\phi(I/K) \times I$, represents the adjustment costs (in terms of lost output). Adjustment costs have the same functional form as Goulder and Hafstead (2017), where net output $\bar{Y} = Y - \phi(I/K) \times I$ and adjustment costs are quadratic in deviations from the level of investment consistent with the steady state growth path, $\phi(I/K) = \frac{\nu(I/K-(\delta+gr))}{I/K}$, where $\nu$ is the primary adjustment cost parameter (equal to two) and $gr$ represents exogenous steady state growth, and $\delta$ is the rate of capital depreciation. The capital stock evolves as $K_{s+1} = I_s - \delta K_s$.

Let $p^k(r)$ denote the cost of capital goods in region $r$. Capital goods are a Leontief aggregate of goods from different sectors; let $\alpha_k^i(r)$ denote the input intensity for good $i$ for region $r$ capital good: $p^k(r) = \sum_i \alpha_k^i(r)p^{inv}_i(r)$ where $p^{inv}_i(r)$ is the net of margin price of purchasing good $i$ in region $r$ for use as a capital good,

$$p^{inv}_i(r) = \frac{p^g_i(r)(1+\sum_m m_{mr}^{inv} p^{m}_m(r))}{1+\sum_m m_{mr}^{inv}}$$ (10)

Let $p_{fi}^i(r)$ denote the quantity of good $i$ from region $r$ demanded for capital goods; $p_{fi}^i(r) = \alpha_k^i \sum_j I_j(r)/(1+\sum_m m_{mr}^{inv})$.

1.3.3 Profits and Behavior of Firms

Firms choose inputs and capital to maximize the value of the firm, $VV$. The value of the firm is the discounted flow of dividends $DIV$ over time, which are functions of after-tax profits $\pi$ and investment costs,

$$DIV_i(r) = \pi_i(r) - p^k(r)I_i(r)$$ (11)

Firms face capital taxes on their profits, but are allowed to deduct depreciation

$$\pi_i(r) = (1 - \tau_k(r))\pi^b_i(r) + \tau_k(r)\delta K^{d}_i(r)$$ (12)

where $\pi^b$ represents capital income (or before tax profits), $\tau_k(r)$ is sum of federal and regional
capital taxes in region \( r \), and \( K^d \) is depreciated capital stock base. Capital income is

\[
\pi^b_i(r) = (1 - \tau_y)p^b_i(r)y_i(r) - p^nr_i(r)nr_i(r) - p^z_iZ_i(r),
\]

(13)

where \( p^b \) is the unit price of output (see below) and \( \tau_y \) is a tax on output (collected by regional governments as proxy for sales and property taxes).

Firms must offer a rate of return (in terms of dividends and capital gains, which equals the change in the value of the firm) consistent with the rate of return from other assets,

\[
DIV_t + VV_{t+1} - VV_t = \tilde{r}_t VV_t.
\]

(14)

In equilibrium, the households’ wealth in region \( r \) \( W_t(r) \) must equal the sum total of the value of firms in region \( r \) such that \( W_t(r) = \sum_i VV_t(i, r) \) and that \( \sum_i DIV_t(i, r) = (1 + \tilde{r}_t)W_t(r) - W_{t+1}(r) \).

### 1.3.4 Optimal Production and Investment

The firm’s Lagrangian is, ignoring sector and regional subscripts,

\[
L = \sum_{s=t}^{\infty} \left[ (1 - \tau_k)[p^b_s(1 - \tau_y)F(K_s, NR_s, Z_s) - p^nr_snr_s - p^z_sZ_s \right.
\]

\[
+ \tau_k \delta K^d_s] - p^k_sI_s - \phi I_s \right] dt(s)
\]

\[
- \sum_{s=t}^{\infty} \lambda^k_s [K_{s+1} - (1 - \delta)K_s - I_s] dt(s)
\]

\[
- \sum_{s=t}^{\infty} \lambda^d_s [K^d_{s+1} - (1 - \delta)K^d_s - p^k_sI_s] dt(s),
\]

(15)

and the first-order conditions are

\footnote{The current version of the model assumes that households only own firms in their own region. Future versions will relax this assumption.}
\[ \frac{\partial L}{\partial Z_s} : (1 - \tau_y) p^y_s f_{Z,s} = p^*_s \]  
(16)

\[ \frac{\partial L}{\partial N_{R,s}} : (1 - \tau_y) p^y_s f_{N_{R,s}} = p^r_{NR,s} \]  
(17)

\[ \frac{\partial L}{\partial I_s} : p_K^s (1 - \lambda d_s) + (1 - \tau_k)(1 - \tau_y) p^y_s \left[ \phi_s + \phi_I^s \frac{I_s}{K_s} \right] = \lambda_s \]  
(18)

\[ \frac{\partial L}{\partial K_{s+1}} : \lambda_s [1 + \tau_{s+1}] = \lambda_{s+1} (1 - \delta) + \left[ (1 - \tau_k)(1 - \tau_y) p^y_{s+1} f_{K,s+1} + (1 - \tau_k)(1 - \tau_y) p^y_{s+1} \phi_I^s \frac{(I_s}{K_s} \right) ]^2 \]  
(19)

Technically, firms cannot choose natural resources, since they are fixed and exogenous. However, equation (17) determines the equilibrium price resource owners receive.

1.3.5 Optimal Supply

DR-GEM uses a Constant Elasticity of Transformation (CET) function to determine the supply of goods across markets. Firms can choose to supply their own regional market, the national market, the foreign market. Firms can also use their goods to supply the market for margins (transportation and trade sectors only). The unit output price \( p^y_i (r) \) is given by

\[ p^y_i (r) = \left[ \alpha^x_y (p_i (r))^{1+\sigma_x} + \alpha^y_i (p^y_i (r))^{1+\sigma_x} + \alpha^f_i (p_i (r))^{1+\sigma_x} + \alpha^m_i \left( p^m_i (r) \right)^{1+\sigma_x} \right]^{\frac{1}{1+\sigma_x}} \]  
(20)

where \( \alpha^x_y \) is a share parameter for market \( x \) and \( \sigma_s \) is the elasticity of transformation that determines the flexibility of producers to supply different markets. The price \( p^m_i (r) \) is the prices received by producers in sector \( i \) for supplying the margin market.

The optimal supply of good \( i \) from region \( r \) to market \( x \), \( y^x_i (r) \) is

\[ y^x_i (r) = (y_i (r) + g^f_i (r) + g^r_i (r)) \alpha^x_y \left( \frac{p^y_i (r)}{p^r_i (r)} \right)^{\sigma_x} \]  
(21)
where $gy_f^i (r)$ and $gy_r^i (r)$ represent government production of good $i$ in region $r$ (see below for details).

1.4. Government

The model uses representative governments for the federal and regional governments. The governments levy taxes and use that revenue to finance government spending on goods and services, labor, and transfers.

1.4.1 Government Expenditure

The production of government services is assumed to use a fixed amount of capital, intermediate inputs, and labor. Let $lg_f^i (r)$ denote real federal government spending on labor from region $r$ and $lg_r^i (r)$ denote real regional government spending on labor from region $r$. Total government spending on labor is $(1 + \tau_p)wlg$.

Government spending on goods and services does not distinguish between spending for intermediate inputs into the production of government services, due to the assumption of fixed inputs into production. Let $gov_f^i (r)$ denote the amount of spending by the federal government on good $i$ in region $r$ and let $gov_r^i (r)$ denote the amount of spending by the regional government $r$ on good $i$ in region $r$.

As is the case for other demands for goods, the government faces margin requirements for purchases of goods. The price to government $x \in \{ f, r \}$ is

$$pg^x_i (r) = \frac{pg^x_i (r)}{1 + \sum_m mr^{gx}_{im}p^m_i (r)} + \sum_m mr^{gx}_{im}p^m_i (r) \quad (22)$$

and the total level of expenditure to procure $gov^x_i (r)$ is $pg^x_i (r)gov^x_i (r)(1 + \sum_m mr^{gx}_{im})$.

Both governments send transfers to households each period, $GT_f^i (r)$ and $GT_r^i (r)$. The level of transfers is held fixed in real terms. The federal government also sends grants to the regional government $r$ only hires labor from its own region. The good $i$ in region $r$ will be a composite of that region’s good, the national market good, and the foreign good.

6Let $cpi_t$, denote the CPI price index such that $cpi_t = \frac{\sum_r \sum_j cp_{jt} (r) cg_{j0} (r)}{\sum_r \sum_j cp_{jt} (r) cg_{j0} (r)}$, where $0$ denotes the benchmark period. Then $GT_f^{it} (r) = cpi_t GT_f^{i0} (r)$ where $GT_f^{i0} (r)$ is the fixed real level of transfers.
governments, $GG(r)$. Regional grants are generally held fixed in real terms (see government budget constraint section for exception below). \(^9\)

Total government expenditures by the federal government in region \(r\) is

\[
GE^f(r) = (1 + \tau_p)w(r)lg^f(r) + \sum_i [pg^f_i(r)gov^f_i(r)(1 + \sum_m mr^g_i)] + GT^f(r) + GG(r).
\]

(23)

Total government expenditures by the regional government in region \(r\) is

\[
GE^r(r) = (1 + \tau_p)w(r)lg^r(r) + \sum_i [pg^r_i(r)gov^r_i(r)(1 + \sum_m mr^g_i)] + GT^r(r).
\]

(24)

1.4.2 Government Revenue

Both governments can produce and sell a limited amount of goods each period (e.g., education). Let \(gy^f_i(r)\) and \(gy^r_i(r)\) denote the fixed quantity of good \(i\) supplied to region \(r\) from each government. The government receives the same price as private producers, \(p^y_i(r)\). Government output is not subject to output taxes.

The federal government collects taxes through labor income taxes \(\tau^f_l\), payroll taxes \(\tau_p\), capital income taxes \(\tau^f_k\) and the regional governments collect taxes through labor, capital, and output taxes, \(\tau^r_l, \tau^r_k, \tau_y\).

Federal government revenues from region \(r\) are given by

\[
T^f(r) = (\tau^f_l + \tau_p)w(r)\ell(r) + \sum_i [\tau^f_k \pi^f_i(r) - \tau^f_k \delta K^d_i(r)] + \sum_i p^y_i(r)gy^f_i(r)
\]

\[\]

\(^9\)GG_i(r) = GG_0(r)cpi_i
where $\ell(r)$ denotes total labor supply, $\ell(r) = \bar{\ell}(r) - l(r)$.

Regional government revenues from region $r$ are given by

\begin{equation}
T^f(r) = \tau^f w(r) \ell(r) + \sum_i [\tau^k \pi^k_i(r) \delta K^d_i(r)]
+ \sum_i p^h_i(r) g y^*_i(r) + \sum_i \tau^y p^y_i(r) y_i(r)
\end{equation}

1.4.3 Government Budget Constraint

As described in the section on households, $GL$ represents lump-sum taxes on households. In each period, government revenues must equal government expenditures such that

\begin{align}
GL^f(r) + T^f(r) &= GE^f(r) \\
GL^r(r) + T^r(r) &= GE^r(r)
\end{align}

Generally, lump-sum taxes adjust each period to close the government budget constraint.\(^\text{10}\) In policy simulations, alternatives includes reducing policy dividends to adjust for tax base effects such that total dividends are less than gross policy revenues and adjusting grants from the federal government to each region.

1.5. Market Clearing

1.5.1 Regional and National Market Clearing

Total demand for each good $g_i$ must be consistent with final good demands.\(^\text{11}\)

\(^{10}\) DR-GEM assumes that federal lump-sum taxes to each region equalize spending and revenue from that region. There is one federal government budget constraint but $r$ lump-sum taxes. This option adds $r - 1$ constraints. Another option would be to fix lump-sum taxes in $r - 1$ regions and adjust lump-sum taxes in one region to close the federal government budget constraint.

\(^{11}\) Time subscripts ignored for clarity.
\begin{equation}
g_i(r) = pce_i(r) + pf_i(r) + gov_i^f(r) + gov_i^r(r) + \sum_j IO_{ij}(r). \tag{29}
\end{equation}

Given the equations for optimal demand and supply for goods produced in region \( r \) to consumers in region \( r \), the model finds prices \( p_i(r) \) such that

\begin{equation}
y_i^r(r) \geq g_i^r. \tag{30}
\end{equation}

Given the equations for optimal demand and supply for goods in the national market, the model finds prices \( p_i^n \) such that

\begin{equation}
\sum_r y_i^n(r) \geq \sum_r g_i^n(r). \tag{31}
\end{equation}

Given the national market assumption, the term \( ADJ^N(r) \) is the nominal value of the initial domestic trade imbalance for each region \( r \), \( ADJ^N(r) = \sum_i p_i^n(g_i^0(r) - y_i^0(r)) \) (where 0 denotes benchmark values).

### 1.5.2 Margin Market Clearing

Demand for margins is a function of the demand for individual goods and the specific margin requirements for each good, which can vary by demand (i.e., the margin requirement for households may be different from the margin requirement for a specific producer).

The current version of the model includes transportation, wholesale, and retail margin requirements. Let \( map_{im}(r) \) denote the mapping from margins to sectors (i.e., which sectors receive the payments for providing margins). If \( p_{im}^{my} \) is the price a sector \( i \) receives for providing margins, then the price of margins is given by \( p_{im}^m(r) = \sum_i map_{im}(r)p_{im}^{my} \).

Total margin demand for margin \( m \) in region \( r \) is
\[ md_m(r) = \sum_i mr_{im}^c pce_i(r) + \sum_i mr_{im}^{inv} pfi_i(r) + \sum_i mr_{im}^{gov} gov_i^r + \sum_i \sum_j mr_{ijm}^{io} IO_{ij}(r) \] (32)

The margin market is assumed to be national. Given the equations for the optimal demand and supply for margin goods, the model finds prices \( p_{my}^i \) such that

\[ \sum_r y_m^i(r) \geq \sum_r \sum_m mapim(r) md_m(r). \] (33)

Given the national market assumption, the term \( ADJM(r) \) is the nominal value of the initial domestic margin imbalance for each region \( r \) (the difference between the margin demanded from region \( r \) from the national margin market and the margin supplied from region \( r \) to the national margin market), \( ADJM(r) = \sum_i p_{my}^i (\sum_m mapim(r) md_m(r) - y_m^i(r)) \) (where 0 denotes benchmark values).

The term \( ADJ(r) \) in the household’s budget constraint is \( ADJ(r) = ADJN(r) + ADJM(r) \).

### 1.5.3 Foreign Market Clearing

The model assumes that the real value of the balance of payments is held constant each period. Given exogenous foreign prices \( p_{f}^i \) and optimal demands for imports and the optimal supply of exports, the model finds the real exchange rate \( exch \) such that

\[ \sum_i (p_{f}^i / exch)(g_{i0}^f - y_{i0}^f) - \sum_i (p_{f}^i / exch)(g_{i}^f - y_{i}^f) \] (34)

where \( (g_{i0}^f - y_{i0}^f) \) represents the benchmark real balance of payments. In equilibrium, \( BOP \) in the household budget constraint is \( BOP = \sum_i (p_{f}^i / exch)(g_{i0}^f - y_{i0}^f) \).
1.6. Emissions Accounting and Emissions Pricing

1.6.1 Emissions Accounting

The DR-GEM model includes coefficients that convert demand for different fuels by sectors, households, and the government into emissions. The current version of the model (July 2022), only includes energy-related carbon dioxide emissions factors $\mu$. These factors are applied to the downstream purchase of coal, natural gas, and refined petroleum products.

Coefficients are calibrated to match energy-related CO$_2$ emissions from the residential, commercial, industrial, transportation, and electric power sectors by fuel (coal, natural gas, and refined petroleum products). Using state-level emissions data and state-level energy expenditure data from the Energy Information Administration, regional-level coefficients $\mu_{EIA}(f, s, r)$ are derived by dividing fuel-sector emissions by fuel-sector expenditures for each region (either a single state or a collection of states).

A mapping from EIA sectors (residential, commercial, industrial, transportation, and electric power) to energy consumers in DR-GEM (households, different sectors, and government) is used to determine model specific coefficients $\mu$.

For example, $\mu^c_j(r)$ denotes direct emissions from the household purchase of consumer good $j$ in region $r$ (in practice, only $j \in \{\text{natural gas, motor vehicle fuels, home heating oil}\}$ are positive): $\mu^c_{\text{natural gas}}(r) = \mu_{EIA}(\text{natural gas, residential}, r)$; $\mu^s_{ij}$ denotes direct emissions from the purchase of fuel $i$ by sector $j$: $\mu^s_{\text{coal, fossil generation}}(r) = \mu_{EIA}(\text{coal, electric power}, r)$; $\mu^g_i$ denotes direct emissions from the government’s purchase of fuels.$^{12}$

Emissions in DR-GEM are then derived by multiplying the emissions factors by the quantity of fuel demanded by each energy consumer, inclusive of margin requirements (i.e., $e^{CO2}_{CO2}(r) = \mu^c_j(r)cg_j(r)$ or $e^{CO2}_{ij} = \mu^s_{ij}IO_{ij}(1 + \sum m mr_{ijm})$). This is because EIA data on energy expenditures include expenditures on transportation and trade margins. Total emissions $e^{CO2}$ are given by

$$e^{CO2} = \sum_j \mu^c_j(r)cg_j(r) + \sum_i \mu^q_i(gov^f_i(r)(1 + \sum m mr_{im}^o))$$  \hspace{1cm} (35)

$$+ \sum_i \mu^q_i(gov^r_i(r)(1 + \sum m mr_{im}^o) + \sum_i \sum_j \mu^g_{ij}IO_{ij}(1 + \sum m mr_{ijm}^o)$$  \hspace{1cm} (36)

$^{12}$The government purchases of natural gas are assigned the EIA emissions factor for natural gas - commercial and the purchases of refined products are assigned the EIA emissions factor for petroleum products - transportation.
1.6.2 Emissions Pricing

Emissions pricing is introduced as a tax on the direct purchase of coal, natural gas, and refined petroleum products.\(^{13}\) Let \(p^{CO2}(r)\) denote the combined federal and regional price energy-related carbon dioxide emissions.\(^{14}\) The new effective price for good \(j\) is \(cp_j = \sum_i G_{ij}p_c + \mu^e_j(r)p^{CO2}(r)\). For firms, the price of good \(i\) for sector \(j\) in region \(r\) becomes \(p^{io}_{ij}(r) + \mu^s_{ij}(r)p^{CO2}(r)\).

Revenue from emissions pricing flows to the government that imposes the tax (either the federal government or a subset of regional governments) and is an added source of government revenue. The current model only allows for lump-sum rebates or dividends to households.

For regional emissions pricing, lump-sum taxes for the regional government imposing the tax are held fixed and the size of dividends is set to meet the government budget constraint. Because the cost of good and services increases and the tax base shrinks as a result of the implementation of emissions pricing, dividends will generally be less than gross emissions pricing. Federal lump-sum taxes and regional lump-sum taxes in non-pricing regions adjust to balance the appropriate government budget constraint.

For federal emissions pricing, federal grants to each region are used to balance the regional government budget constraint with regional lump-sum taxes held fixed in real terms. Dividends are adjusted to meet the federal budget constraint.

\(^{13}\)Crude oil is purchased only by refiners and no emissions factor is applied to those purchases.

\(^{14}\)In practice, the model allows for fuel-sectors to be omitted from carbon pricing policy through the use of fuel-sector dummy variables. The model also specifies the real price of carbon dioxide, and the price is scaled by the CPI each period. Both were excluded here for clarity.
2. How to use the code

The Dynamic Regional General Equilibrium Model (DR-GEM) is solved in GAMS using the PATH solver. GAMS can be downloaded for free, but the size of EDGE requires licenses for both GAMS and PATH. PATH should be set as the default solver for mcp problems.

2.1. List of programs

The DR-GEM programs are meant to be fully self-contained. A "MAIN" file calls the appropriate subprograms and will complete policy simulations specified within the "MAIN" file (DR-GEM-2022_v1_MAIN_LUMP.gms).

The sections below explain how to execute the program and briefly specifies steps and subprograms:

DR-GEM_datainput_complete.gms file builds the full national social accounting matrix. Then, the model is parameterized using the subprogram DR-GEM_parameters.gms. Afterwards, DR-GEM_wash_US.gms reconciles national data set with chosen parameters and model identities.

Four subprograms are then utilized to create a model consistent state-level social accounting matrix:

- DR-GEM_load_implan_51.gms
- DR-GEM_impose_51.gms
- DR-GEM_SEDS_51_import.gms
- DR-GEM_disagg_51.gms

The next step is to determine model aggregation. For example, DR-GEM_aggregation_i22_r4.gms aggregates the model's benchmark data set into 22 industries and four regions.

Next, the elasticities of substitution must be defined, and then the remaining model parameters can be calibrated to these elasticities and the underlying social accounting matrix.

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15 DR-GEM_datainput_complete.gms includes calls to .dat data sets that are not included in the GitHub folder because they are proprietary.
16 DR-GEM_load_implan_51.gms includes calls to .dat data sets that are not included in the GitHub folder because they are proprietary.
The Dynamic Regional General Equilibrium Model (DR-GEM) is solved in GAMS using the PATH solver.

### 2.2. Interpreting Model Output

The current version of DR-GEM should only be used to calculate the percent change from baseline for various outputs such as emissions or GDP. The model should not be used to project the level of emissions or GDP in any given year.

To formulate a reference case in which it would be appropriate to project level changes in key
model outcomes, key model parameters such as household preferences, supply of natural resources, and production share parameters would need to be exogenously changed to match some external reference case projection for emissions by fuel-sector. An algorithm to implement this in DR-GEM is forthcoming (as of July 2022).
References


URL www.implan.com
Appendix A

Consider the CES composite $X$ made up of goods $X_1, \ldots, X_n$.

$$X = \gamma_x \left[ \sum_{j=1}^{n} \alpha_x X_j^{\rho_x} \right]^{\frac{1}{\rho_x}} \tag{37}$$

where $\sum_{j=1}^{n} \alpha_x = 1$. The parameter $\rho_x$ is a simple translation of the elasticity of substitution $\sigma^x$: $\rho_x = (\sigma^x - 1)/\sigma^x$.

Following a standard cost minimization problem, the unit price of $X$ is

$$\hat{p}^x = (\gamma^x)^{\frac{1}{1-\sigma^x}} \left[ \sum_{j=1}^{n} (\alpha^x_j)^{\sigma^x} (\hat{p}^x_j)^{1-\sigma^x} \right]^{\frac{\sigma^x}{1-\sigma^x}} \tag{38}$$

and the optimal intensity for expenditure is

$$\frac{X_j}{X} = (\gamma^x \alpha^x_j)^{\sigma^x} \left[ \frac{\hat{p}^x_j}{\hat{p}^x} \right]^{-\sigma^x}. \tag{39}$$