$$\boxed{SWF = E\left[\int e^{-\delta t} u(c(t))dt\right]} \longrightarrow \boxed{r_t = \delta - \frac{1}{t}\ln\frac{Eu'(c_t)}{u'(c_0)}}$$

- What is the status of δ, u?
  - Ethical, or preference-based?
  - Concavity of u: aversion to risk, fluctuation, inequity?
- Should we disentangle these three dimensions?
- Functional form of u? $\lim_{c\to 0} u'(c) = +\infty$?
- Link with market prices?
- Calibration of c_t?

The extended Ramsey rule in the iid lognormal case

$$c_{t+1} = c_t e^{x_t}, \text{ with } x_t \text{ i.i.d.} \sim N(\mu, \sigma)$$

$$g = \ln(Ec_1 / c_0) = Ee^x = \mu + 0.5\sigma^2$$

$$\frac{Eu'(c_t)}{u'(c_0)} = \frac{E\left[c_0^{-\gamma} \prod_\tau e^{-\gamma x_\tau}\right]}{c_0^{-\gamma}} = \left[Ee^{-\gamma x}\right]^t = \left[e^{-\gamma(\mu - 0.5\gamma\sigma^2)}\right]^t.$$

$$r_t = \delta - \frac{1}{t}\ln\frac{Eu'(c_t)}{u'(c_0)} = \delta + \gamma\mu - 0.5\gamma^2\sigma^2.$$

$$r = \delta + \gamma g - 0.5\gamma(\gamma+1)\sigma^2.$$

Impatience wealth effect precautionary effect

Interpersonal MRS approach

- Consider an economy with 2 social groups of equal size, A and B. Each agent in group A is 2 times wealthier than in group B.
- We can transfer wealth from A to B. What is the maximum sacrifice of A that Society should accept for B to get one more1€?

γ	MRS 2 wA = 2*wB	MRS 10 wA = 10*wB
0	1,00	1,00
0,5	1,41	3,16
1	2,00	10,00
1,5	2,83	31,62
2	4,00	100,00
4	16,00	10000,00

Certainty equivalent approach

- You are indifferent between
  - 50-50 chance to live with a daily income of 80 or 120;
  - A sure daily income of X.

γ	Certainty equiv (80,1/2;120,1/2)	Certainty equiv (50,1/2;150,1/2)
0	100,00	100,00
0,5	98,99	93,30
1	97,98	86,60
1,5	96,98	80,38
2	96,00	75,00
4	92,44	62,24

- Risk aversion or aversion to inequity (veil of ignorance).

Standard time-series calibration of the extended Ramsey rule

- Kocherlakota (1996), using United States annual data over the period 1889-1978, estimated the standard deviation of the growth of consumption per capita to 3.6% per year.

$$\sigma^2 = (0.036)^2 \text{ and } \gamma = 2 \text{ implies } 0.5\gamma(\gamma+1)\sigma^2 = 0.4\%.$$

- Benchmark calibration

g	σ	δ	γ
2%	*3.6%*	*0%*	*2*

$$r = \delta + \gamma g - 0.5\gamma(\gamma+1)\sigma^2 = 3.6\%$$

Calibration of the Ramsey rule

	Country	g (wealth effect)	σ (precautionary effect)	Discount rate
Developed countries	United States	1.74% *(3.48%)*	2.11% *(-0.13%)*	*3.35%*
	France	1.75% *(3.50%)*	1.57% *(-0.07%)*	*3.43%*
	Germany	1.76% *(3.52%)*	1.83% *(-0.10%)*	*3.42%*
	United Kingdom	1.86% *(3.71%)*	2.18% *(-0.14%)*	*3.57%*
	Japan	2.34% *(4.67%)*	2.61% *(-0.20%)*	*4.47%*
Emerging countries	China	7.60% *(15.20%)*	3.53% *(-0.37%)*	*14.82%*
	South Korea	5.38% *(10.75%)*	3.40% *(-0.35%)*	*10.41%*
	Taiwan	5.41% *(10.82%)*	5.29% *(-0.84%)*	*9.98%*
	India	3.34% *(6.88%)*	3.03% *(-0.28%)*	*6.61%*
	Russia	1.54% *(3.08%)*	5.59% *(-0.94%)*	*2.14%*
Africa	Gabon	1.29% *(2.58%)*	9.63% *(-2.78%)*	*-0.20%*
	Liberia	-1.90% *(-3.79%)*	19.58% *(-11.50%)*	*-15.30%*
	Zaire (RDC)	-2.76% *(-5.53%)*	5.31% *(-0.85%)*	*-6.38%*
	Zambia	-0.69% *(-1.38%)*	4.01% *(-0.48%)*	*-1.86%*
	Zimbabwe	-0.26% *(-0.53%)*	6.50% *(-1.27%)*	*-1.79%*

Table 1: Country-specific discount rate computed from the extended Ramsey rule using the historical mean g and standard deviation σ of growth rates of real GDP/cap 1969-2010.

Calibration of the Ramsey rule (Ct'd)

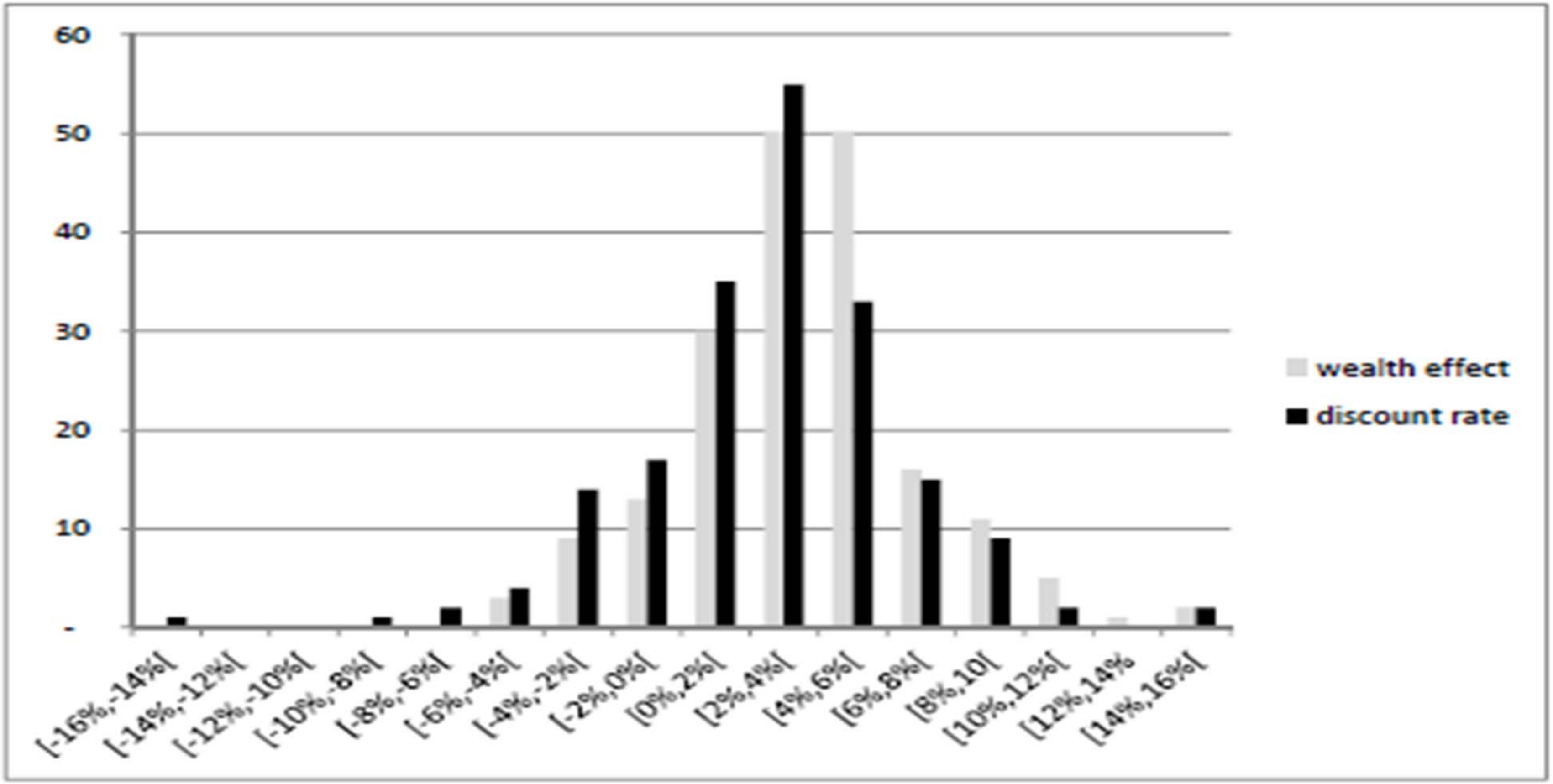
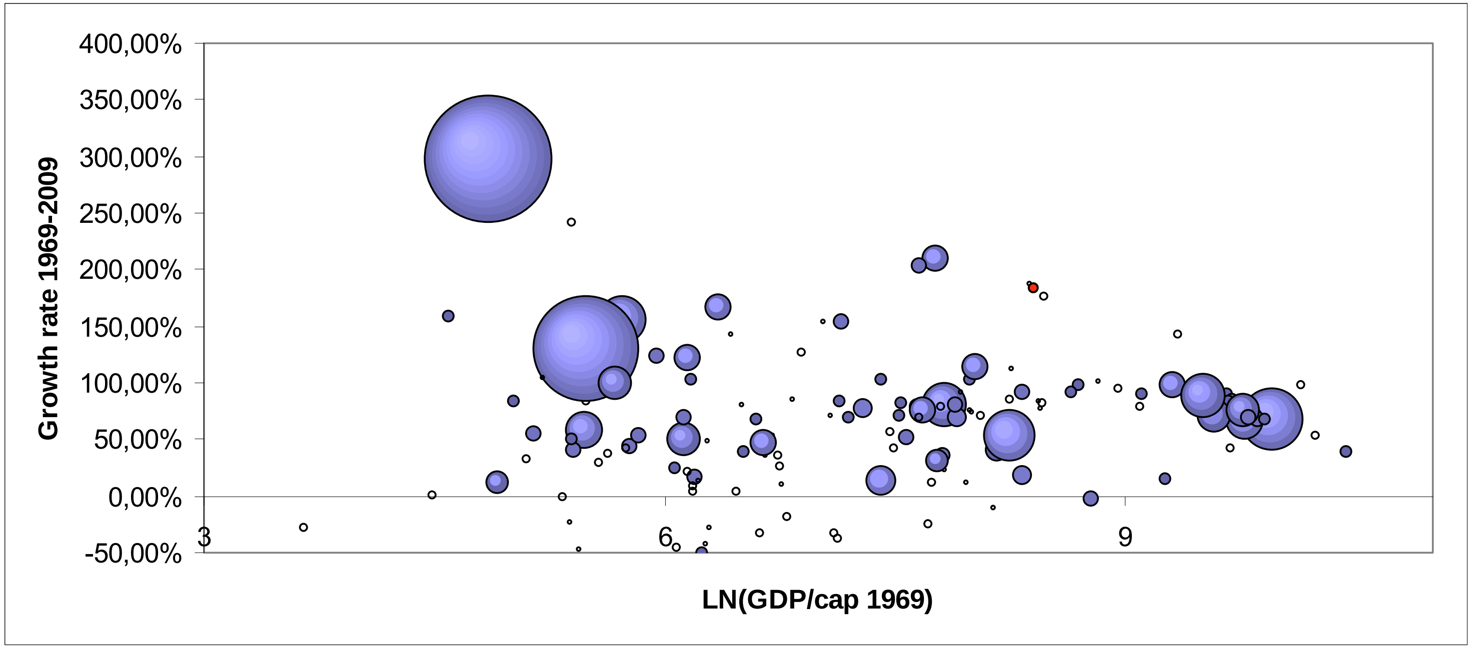


Figure 1: Frequency for the wealth effect and the discount rate among the 190 countries, using the extended Ramsey rule.

Alternative ross-sectional)calibration of the extended Ramsey rule

- 190 countries over the period 1969-2009:



μ	σ	δ	γ
1.5%	*11%*	*0%*	*2*

$$
\begin{aligned}
r &= \delta + \gamma g - 0.5\gamma(\gamma+1)\sigma^2 \\
&= 0\% + 3\% - 2.42\% \\
&= 0.58\%
\end{aligned}
$$

Pricing the future
The economics of discounting and sustainable development

Markov switches

Christian Gollier

Toulouse School of Economics

Two-regime Markov process

$$\begin{cases} c_{t+1} = c_t e^{x_t} \\ x_t = \mu^{s_t} + \varepsilon_t \\ P\left[s_{t+1} = b \middle| s_t = g\right] = \pi^g; \quad P\left[s_{t+1} = g \middle| s_t = b\right] = \pi^b \end{cases}$$

$$\frac{E\left[u'(c_1) \middle| s\right]}{u'(c_0)} = (1-\pi^s)Ee^{-\gamma(\mu^s+\varepsilon_0)} + \pi^s Ee^{-\gamma(\mu^{-s}+\varepsilon_0)} = e^{0.5\gamma^2\sigma^2}\left[(1-\pi^s)e^{-\gamma\mu^s} + \pi^s e^{-\gamma\mu^{-s}}\right].$$

$$r_1^s = \delta + \gamma m_1^s - 0.5\gamma^2\sigma^2,$$

$$e^{-\gamma m_1^s} = (1-\pi^s)e^{-\gamma\mu^s} + \pi^s e^{-\gamma\mu^{-s}}.$$

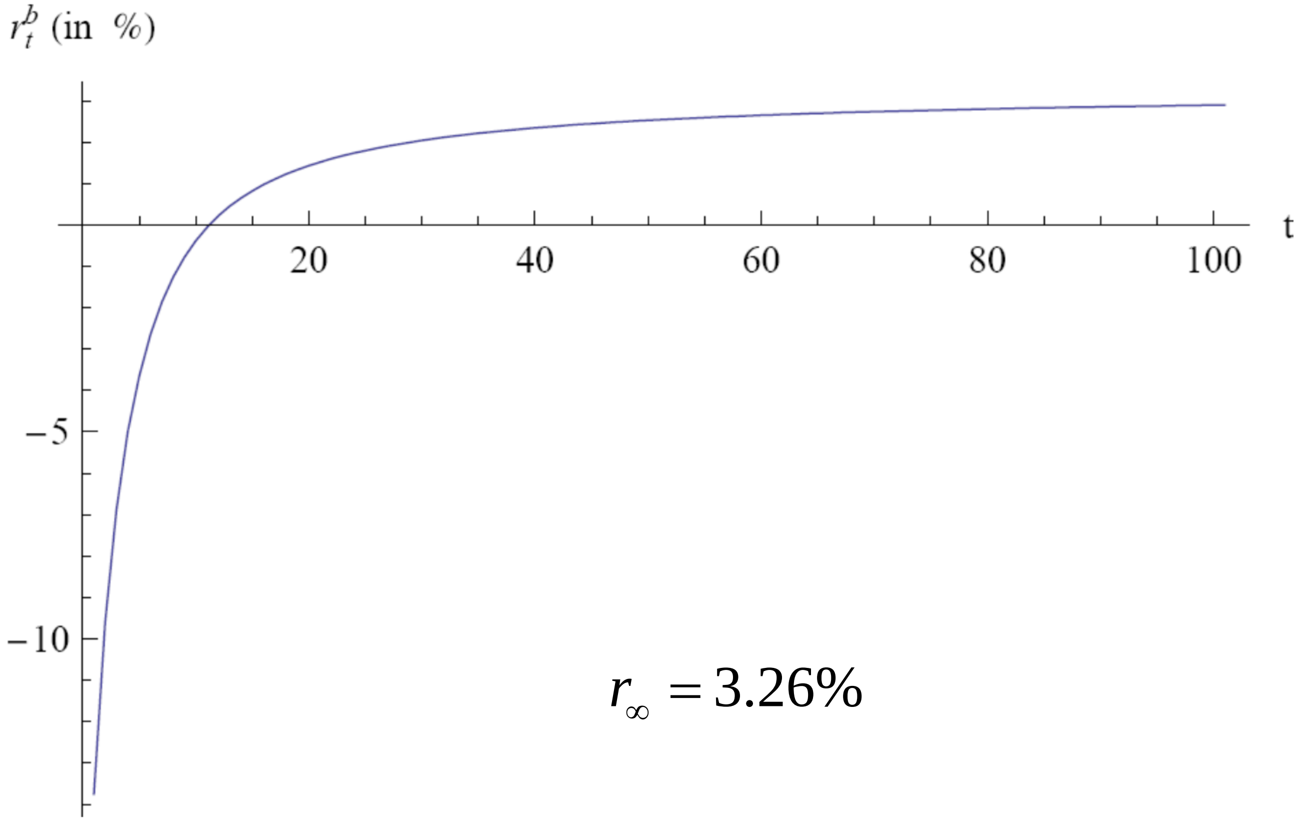
Precautionary equivalent growth rate

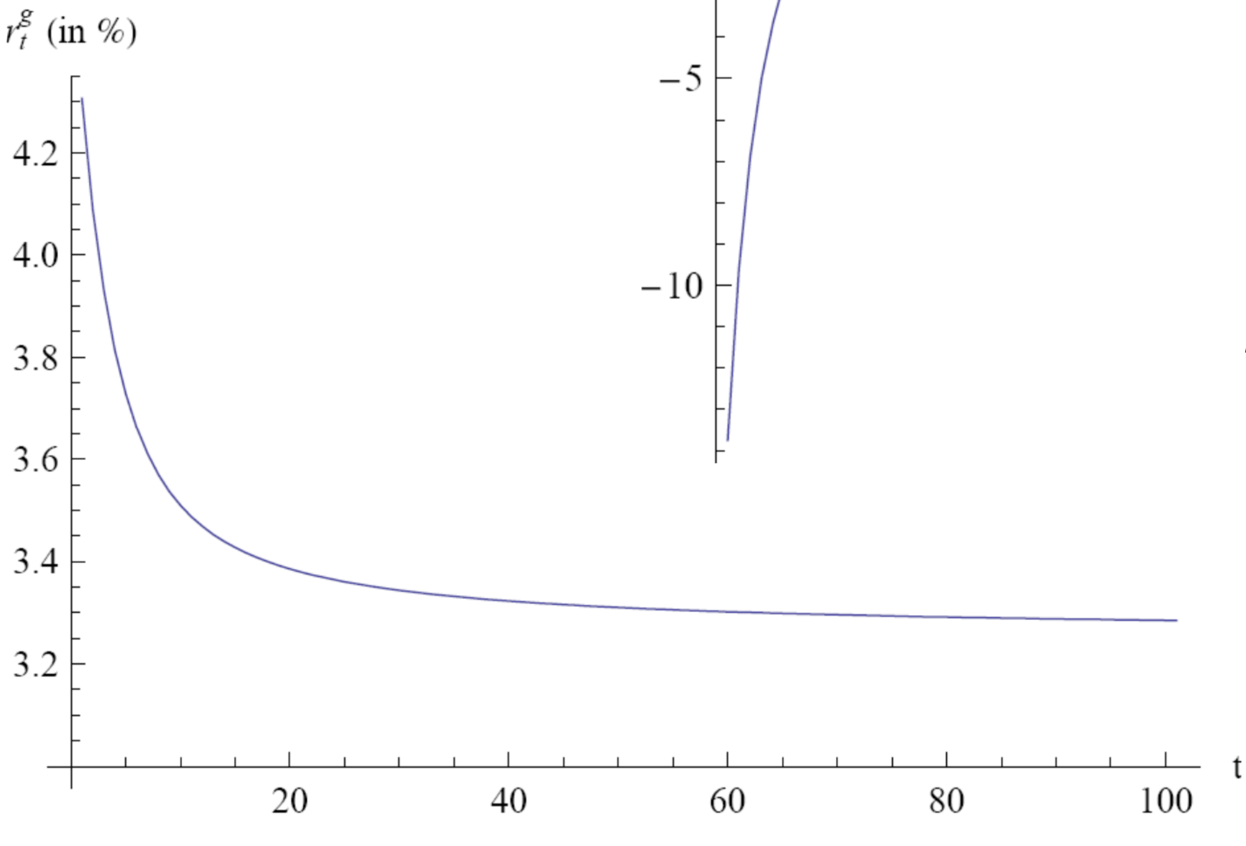
$$m_1^g \geq m_1^b$$

Numerical sim I

- Link with the literature on extreme events (Rietz (1988), Aase (1993), Barro (2006)).
- Cecchetti, Lam and Mark (2000) estimated a two-state regime-switching process for the US economy using the annual per capita consumption data covering the period 1890-1994.
- The unconditional expected growth rate is *1.89%*.

μ^g	μ^b	π^g	π^b	σ
2.25%	−6.78%	2.2%	48.4%	3.13%





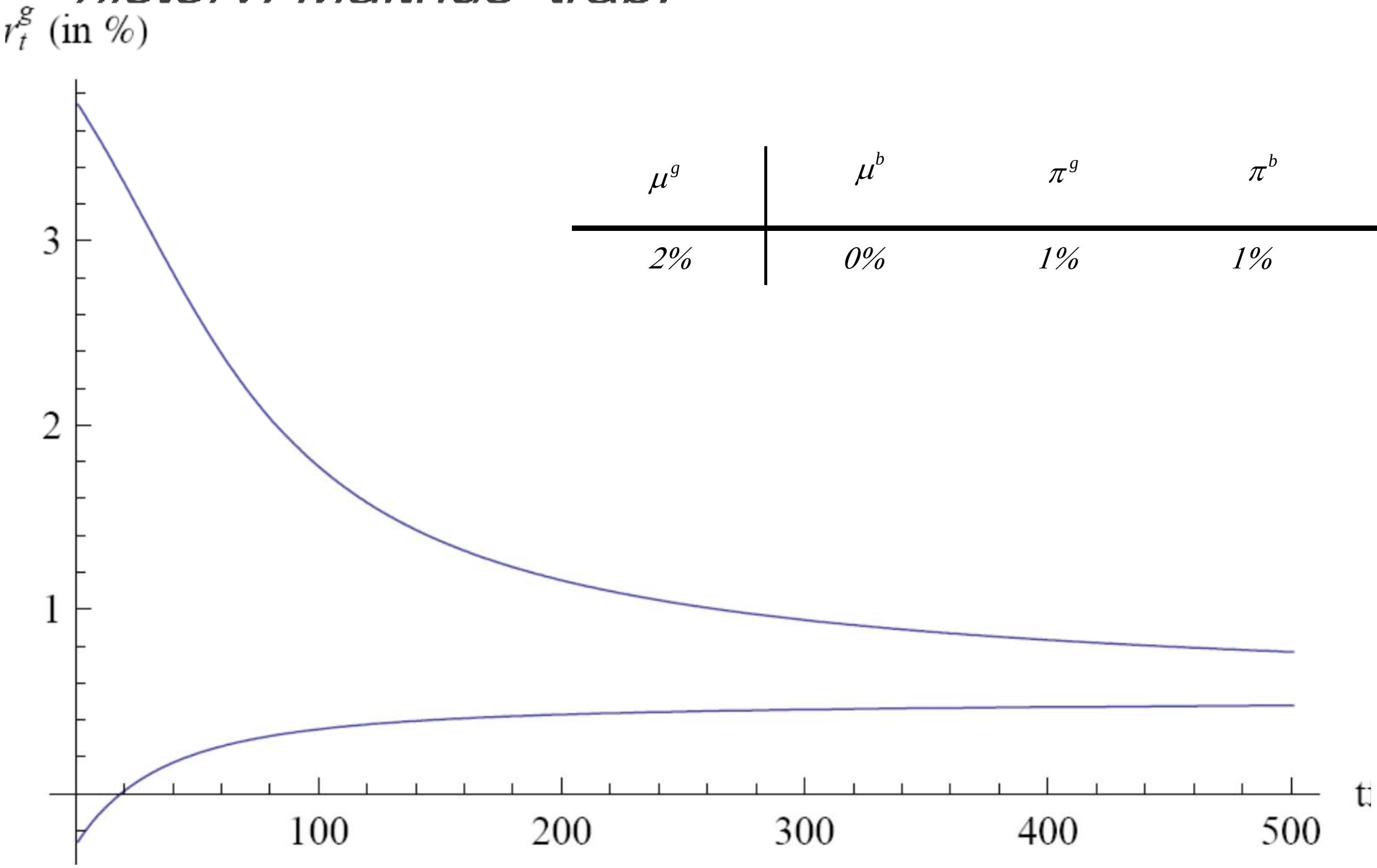
$$r_\infty = 3.26\%$$

Persistent shocks on the growth rate

- Daily wage (in pounds of wheat):
  - In Babylon (1880-1600 B.C.): around 15;
  - In the golden age of Pericles in Athens: around 26;
  - In England around 1780: 13.
- Malthus Law? Stable 0% growth of GDP/cap.
- Switch to a trend of 2% around 1800-1850.

Numerical sim II

- The calibration based on data covering the period 1890-1994 fails to recognize a crucial aspect of economic history: Malthus' trap.



μ^g	μ^b	π^g	π^b	σ
2%	0%	1%	1%	3.6%

Pricing the future: The economics of discounting and sustainable development

Parametric uncertainty and fat tails

Christian Gollier

Toulouse School of Economics

Uncertain growth

- Dynamic process on c_t parametrized by θ.
- $\theta=1,\ldots,n$ with probabilities $q_1,q_2,\ldots,q_n$.
- By the law of iterated expectations, we have that

$$Eu'(c_t)=\sum_{\theta=1}^{n} q_\theta E\left[u'(c_t)\middle|\theta\right].$$

$$r_t=\delta-\frac{1}{t}\ln\sum_{\theta=1}^{n} q_\theta \frac{E\left[u'(c_t)\middle|\theta\right]}{u'(c_0)}=-\frac{1}{t}\ln\sum_{\theta=1}^{n} q_\theta e^{-r_{t\theta}t}$$

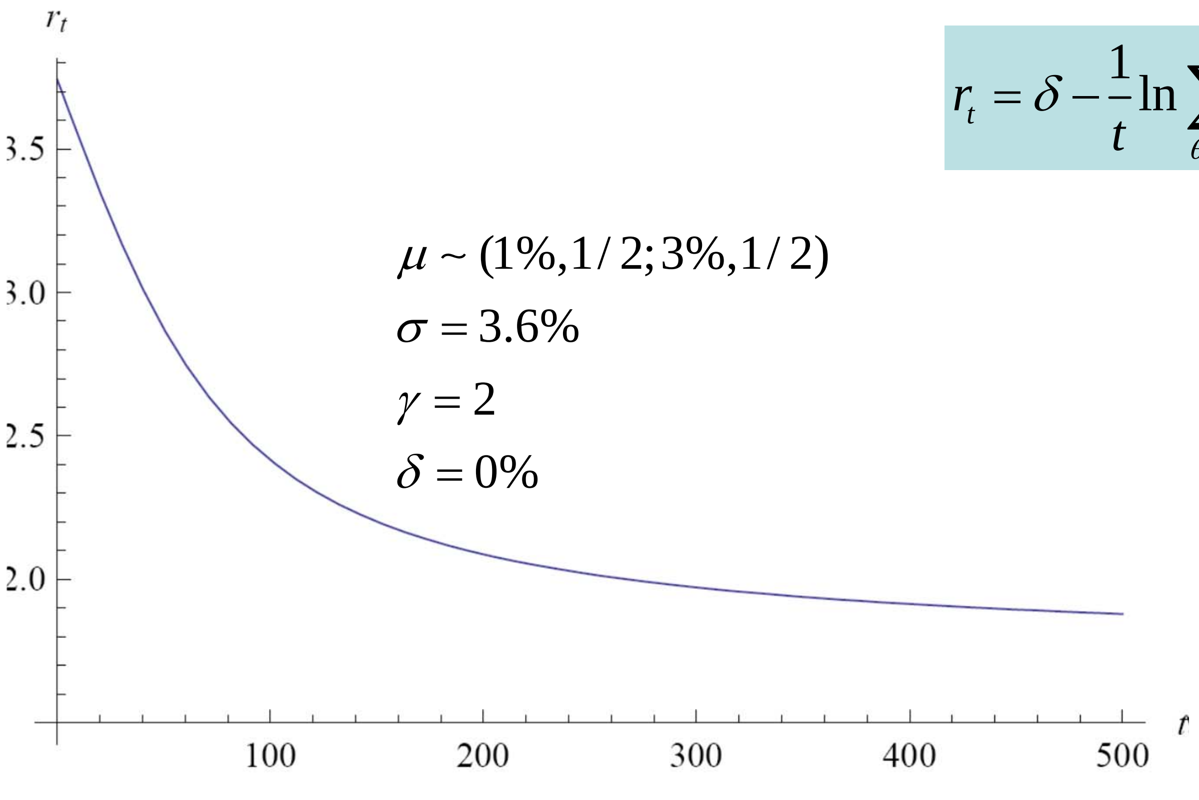
$$r_{t\theta}=\delta-\frac{1}{t}\ln\frac{E\left[u'(c_t)\middle|\theta\right]}{u'(c_0)}$$

Conditional to θ, the growth process is a random walk

$$\begin{cases} c_{t+1} = c_t e^{x_t} \\ x_0, x_1, \ldots | \theta \ i.i.d. \sim N(\mu_\theta, \sigma_\theta) \ \forall \theta \\ \theta \sim (1, q_1; \ldots; n, q_n) \end{cases}$$

$$r_{t\theta} = \delta + \gamma\mu_\theta - 0.5\gamma^2\sigma_\theta^2.$$

$$r_t = \delta - \frac{1}{t}\ln\sum_{\theta=1}^{n} q_\theta e^{(-\gamma\mu_\theta + 0.5\gamma^2\sigma_\theta^2)t}.$$



$\mu \sim (1\%, 1/2; 3\%, 1/2)$

$\sigma = 3.6\%$

$\gamma = 2$

$\delta = 0\%$

The case of an unknown trend of economic growth

- Suppose that σ is known, but μ is normally distributed with mean μ_0 and std deviation σ_0.

$$r_t = \delta - \frac{1}{t}\ln e^{(-\gamma\mu_0 + 0.5\gamma^2 t\sigma_0^2 + 0.5\gamma^2\sigma^2)t} = \delta + \gamma\mu_0 - 0.5\gamma^2(\sigma^2 + \sigma_0^2 t).$$

$$\left.\begin{aligned} \ln\frac{c_t}{c_0}\Big|\mu,\sigma &\sim N(\mu t, \sigma^2 t) \\ \mu t &\sim N(\mu_0 t, \sigma_0^2 t^2) \end{aligned}\right\} \Rightarrow \ln\frac{c_t}{c_0} \sim N(\mu_0 t, \sigma^2 t + \sigma_0^2 t^2)$$

$$\min r_\theta = -\infty$$

The case of an unknown volatility of economic growth

- Weitzman (2007, 2009) : Suppose alternatively that μ is known, but σ is not.
- We work with the precision $p_\theta = \sigma_\theta^{-2} \sim \Gamma(a,b).$
- Unconditional distribution of x_t:

$$\left.\begin{array}{c} x \,\middle|\, p \sim N(\mu, \sigma = 1/\sqrt{p}) \\ p \sim \Gamma(a,b) \end{array}\right\} \Rightarrow \frac{x-\mu}{1/\sqrt{ab}} \sim Student(2a)$$

- As is well-known also, this Student's t-distribution has fatter tails than the corresponding normal distribution with the same mean and variance.

$$r_t = \delta - \frac{1}{t}\ln\frac{Eu'(c_t)}{u'(c_0)} = -\infty$$